Strong coupling constants of bottom and charmed mesons with scalar, pseudoscalar, and axial vector kaons

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The strong coupling constants, $g_{D_sDK_0^*}$, $g_{B_sBK_0^*}$, $g_{D_s^*DK}$, $g_{B_s^*BK}$, $g_{D_s^*DK_1}$ and $g_{B_s^*BK_1}$, where K_0^*, K and K_1 are scalar, pseudoscalar, and axial-vector kaon mesons, respectively, are calculated in the framework of three-point QCD sum rules. In particular, the correlation functions of the considered vertices when both $B(D)$ and $K_0^*(K)(K_1)$ mesons are off shell are evaluated. In the case of K_1, which is either $K_1(1270)$ or $K_1(1400)$, the mixing between these two states are also taken into account. A comparison of the obtained result with the existing prediction on $g_{D_s^*DK}$ as the only coupling constant among the considered vertices, previously calculated in the literature, is also made.

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I. INTRODUCTION

The strong coupling constants among the bottom and charmed mesons with light scalar, pseudoscalar, and axial strange mesons are the main ingredients in analysis of their strong interactions. More accurate determination of these coupling constants is needed to better understand the strong interactions among the participated mesons, construct the strong potentials among them, and obtain knowledge about the nature and structure of the encountered particles. Experimentally, it is believed that in the production of the charmonium states like J/ψ and ψ' from the B_c or newly discovered charmonium X, Y, and Z states by the *BABAR* and BELLE collaborations, there are intermediate two body states containing D, D_s, D^* and D_s^* mesons (for example, the kaon can annihilate the charmonium in a nuclear medium to give D and D_s mesons), which decay to the final J/ψ and ψ' states exchanging one or more virtual mesons. A similar story would happen in decays of heavy bottonium. To exactly follow and analyze the procedure in the experiment, we need to knowledge about the coupling constants among the particles involved.

The strong coupling constants among mesons take place in low energies very far from the perturbative region, where the strong coupling constant between quarks and gluons is large and perturbation theory fails. Hence, in the hadronic scale, one should consult to some nonperturbative methods in QCD to describe nonperturbative phenomena. Among the nonperturbative methods, the QCD sum rules approach [1–4] is one of the most powerful, applicable, and attractive one as it is based on QCD Lagrangian and is free of a model dependent parameter. This approach has rendered many successful predictions such as its predictions about the vector mesons [5–9]. The three-point correlation function has been widely used to calculate many parameters of hadrons (see for instance [10–13]). The QCD sum rules for some strong coupling constants were derived by means of the three-point functions in [14]. In the present work, we investigate various strong coupling constants among bottom (charmed)-bottom strange (charmed strange) mesons with scalar, pseudoscalar, and axial-vector kaons. Calculation of such coupling constants can help us in understanding the nature of the strong interaction among the participating particles.

In the case of the scalar kaon, we consider the $B_s - B - K_0^*$ and $D_s - D - K_0^*$ vertices for both $K_0^*(800)$ and $K_0^*(1430)$. Understanding the internal structure of the scalar mesons has been a striking issue in the last 30–40 years. Despite their investigation both theoretically and experimentally, most of their properties are not very clear yet. Detection and identification of the scalar mesons are difficult, experimentally, so the theoretical and phenomenological works can play a crucial role in this regard. In this work, we also calculate the coupling constants $g_{B_s^*BK}$ and $g_{D_s^*DK}$ for pseudoscalar K. The next aim in the present work is to consider the vertices $B_s^* - B - K_1$ and $D_s^* - D - K_1$ for both $K_1(1270)$ and $K_1(1400)$ axial states taking into account their mixture.

Experimentally, the $K_1(1270)$ and $K_1(1400)$ are the mixtures of the strange members of two axial-vector SU(3) octets $^3P_1(K_1^A)$ and $^1P_1(K_1^B)$. To avoid any confusion between the B meson and the sign B in the K_1^B, we will use the $K_1^{a(b)}$ instead of $K_1^{A(B)}$ in this article. The $K_1(1270, 1400)$ are related to the $K_1^{a,b}$ states via [15,16]

$$
\begin{aligned}
|K_1(1270)\rangle &= |K_1^a\rangle \sin\theta + |K_1^b\rangle \cos\theta,\\
|K_1(1400)\rangle &= |K_1^a\rangle \cos\theta - |K_1^b\rangle \sin\theta,
\end{aligned}
\tag{1}
$$

where the mixing angle θ takes the values in the interval $37° \leq \theta \leq 58°$, $-58° \leq \theta \leq -37°$ [15–19]. The sign

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ambiguity for the mixing angle is correlated with the fact that one can add arbitrary phase to the $|K_1^a\rangle$ and $|K_1^b\rangle$. Studies on $B \to K_1(1270)\gamma$ and $\tau \to K_1(1270)\nu_\tau$ lead to the following value for θ [20]:

$$\theta = -(34 \pm 13)°. \tag{2}$$

In the present work, contributing the quark-quark and quark-gluon condensate diagrams as nonperturbative effects, we evaluate the corresponding correlation functions when both $B(D)$ and $K_0^*(K)(K_1)$ mesons are off shell. Note that recently, we have investigated the $D_s^* D K^*(892)$, and $B_s^* B K^*(892)$ vertices for K^* being the vector meson in the framework of the three-point QCD sum rules in [21]. Moreover, the following coupling constants have been investigated via three-point and light cone QCD sum rules in the literature: $D^* D\pi$ [22,23], $DD\rho$ [24], DDJ/ψ [25], $D^* DJ/\psi$ [26], $D^* D^* \pi$ [27,28], $D^* D^* J/\psi$ [29], $D_s D^* K$, $D_s^* DK$ [27,30], $D_0 D_s K$, $D_{s0} DK$ [31], $DD\omega$ [32], $D^* D^* \rho$ [33], $D^* D\rho$ [34], $B_{s0} BK$, $B_{s1} B^* K$ [35], $B_{s0} BK$ [36], $a_0 \eta \pi^0$, $a_0 \eta' \pi^0$ [37], $a_0 K^+ K^-$ [38] and $f_0 K^+ K^-$ [38,39].

The outline of the paper is as follows. In Sec. II, we introduce the responsible correlation functions, and we obtain QCD sum rules for the strong coupling constant of the considered vertices. For each of the scalar, pseudoscalar, and axial kaon cases, we will calculate the correlation function when both the $B(D)$ and $K_0^*(K)(K_1)$ mesons are off shell. In the case of the K_1 meson, first we will calculate the QCD sum rules for the vertices $B_s^* - B - K_1^a$ and $D_s^* - D - K_1^b$, then using the relations in Eq. (1), we will acquire the QCD sum rules for the vertices $B_s^* - B - K_1(1270)$ and $D_s^* - D - K_1(1400)$. In obtaining the sum rules for physical quantities, we will consider both light quark-light quark and light quark-gluon condensate diagrams as nonperturbative contributions. Finally, in Sec. III, we numerically analyze the obtained sum rules for the considered strong coupling constants. We will obtain the numerical values for each coupling constant when both the $B(D)$ and $K_0^*(K)(K_1)$ states are off shell. Then taking the average of the two off-shell cases, we will obtain final numerical values for each coupling constant. In this section, we also compare our result on $g_{D_s^* DK}$ with existing prediction in the literature.

II. QCD SUM RULES FOR THE STRONG COUPLING CONSTANTS

In this section, we obtain QCD sum rules for the strong coupling constants associated with the $D_s(B_s) - D(B) - K_0^*$, $D_s^*(B_s^*) - D(B) - K$, and $D_s^*(B_s^*) - D(B) - K_1$ vertices. We start our discussion considering the sufficient correlation functions responsible for the corresponding strong transition involving each K_0^*, K and K_1 mesons when both $D(B)$ and $K_0^*(K)(K_1)$ are off shell. The following three-point correlation functions describe the considered strong transitions:

(i) correlation functions corresponding to the $D_s(B_s) - D(B) - K_0^*$ vertex:
for $D(B)$ off shell:

$$\Pi^{D(B)} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta^{K_0^*}(x)\eta^{D(B)}(y) \times \eta^{D_s(B_s)\dagger}(0))|0\rangle, \tag{3}$$

for K_0^* off shell:

$$\Pi^{K_0^*} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta^{D(B)}(x)\eta^{K_0^*}(y) \times \eta^{D_s(B_s)\dagger}(0))|0\rangle, \tag{4}$$

(ii) correlation functions corresponding to the $D_s^*(B_s^*) - D(B) - K$ vertex:
for $D(B)$ off shell:

$$\Pi_\mu^{D(B)} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta^{K}(x)\eta^{D(B)}(y) \times \eta_\mu^{D_s^*(B_s^*)\dagger}(0))|0\rangle, \tag{5}$$

for K off shell:

$$\Pi_\mu^{K} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta^{D(B)}(x)\eta^{K}(y) \times \eta_\mu^{D_s^*(B_s^*)\dagger}(0))|0\rangle, \tag{6}$$

(iii) correlation functions corresponding to the $D_s^*(B_s^*) - D(B) - K_1$ vertex:
for $D(B)$ off shell:

$$\Pi_{\mu\nu}^{D(B)} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta_\nu^{K_1}(x)\eta^{D(B)}(y) \times \eta_\mu^{D_s^*(B_s^*)\dagger}(0))|0\rangle, \tag{7}$$

$$\Pi_{\mu\nu\nu'}^{D(B)} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta_{\nu\nu'}^{K_1}(x)\eta^{D(B)}(y) \times \eta_\mu^{D_s^*(B_s^*)\dagger}(0))|0\rangle, \tag{8}$$

for K_1 off shell:

$$\Pi_{\mu\nu}^{K_1} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta^{D(B)}(x)\eta_\nu^{K_1}(y) \times \eta_\mu^{D_s^*(B_s^*)\dagger}(0))|0\rangle, \tag{9}$$

$$\Pi_{\mu\nu\nu'}^{K_1} = i^2 \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0| \mathcal{T}(\eta^{D(B)}(x)\eta_{\nu\nu'}^{K_1}(y) \times \eta_\mu^{D_s^*(B_s^*)\dagger}(0))|0\rangle, \tag{10}$$

where $\mathcal{T}$ is the time ordering product, q is the momentum of the off-shell state, p' is the momentum of the final on-shell state. We will set the momentum of the initial state as $p = p' + q$. In the vertex containing K_1 meson, we have two correlation functions for both off-shell cases since this meson couples into two interpolating currents $\eta_\nu^{K_1}$ and $\eta_{\nu\nu'}^{K_1}$. We will define these couplings in terms of G-parity conserving and G-parity violating decay constants later. The interpolating currents, which produce the considered mesons from the vacuum with the same quantum numbers as the interpolating currents can be written in terms of the quark field operators as following form:

$$\begin{aligned}
\eta^{K_0^*}(x) &= \bar{s}(x)Uu(x),\\
\eta^{K}(x) &= \bar{s}(x)\gamma_5 u(x),\\
\eta_\nu^{K_1}(x) &= \bar{s}(x)\gamma_\nu\gamma_5 u(x),\\
\eta_{\nu\nu'}^{K_1}(x) &= \bar{s}(x)\sigma_{\nu\nu'}\gamma_5 u(x),\\
\eta^{D(B)}(x) &= \bar{u}(x)\gamma_5 c(b)(x),\\
\eta^{D_s(B_s)}(x) &= \bar{s}(x)\gamma_5 c(b)(x),\\
\eta_\mu^{D_s^*(B_s^*)}(x) &= \bar{s}(x)\gamma_\mu c(b)(x),
\end{aligned} \tag{11}$$

where U stands for unit matrix and u, s, c and b are the up, strange, charm, and bottom quark fields, respectively.

According to general philosophy of the QCD sum rules, we calculate the aforementioned correlation functions in two different representations. In physical or phenomenological representation, we calculate them in terms of hadronic parameters. In QCD or theoretical representation, we evaluate them in terms of QCD degrees of freedom like quark masses, quark condensates, etc. with the help of the operator product expansion, where the perturbative and nonperturbative contributions are separated. The QCD sum rules for strong coupling constants are obtained equating these two different representations through dispersion relation. To suppress contributions of the higher states and continuum, we will apply double Borel transformation with respect to the momentum squared of the initial and final on-shell states to both sides of the obtained sum rules.

First, let us focus on the calculation of the physical sides of the aforesaid correlation functions, for example, when $D(B)$ meson is off shell. Saturating the correlation functions with the complete sets of three participating particles and isolating the ground states and after some straightforward calculations, we obtain:

(i) physical representation corresponding to the $D_s(B_s) - D(B) - K_0^*$ vertex:

$$\Pi^{D(B)} = \frac{\langle 0|\eta^{K_0^*}|K_0^*(p')\rangle\langle 0|\eta^{D(B)}|D(B)(q)\rangle\langle K_0^*(p')D(B)(q)|D_s(B_s)(p)\rangle\langle D_s(B_s)(p)|\eta^{D_s(B_s)}|0\rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s(B_s)}^2)(p'^2 - m_{K_0^*}^2)} + \ldots, \tag{12}$$

(ii) physical representation corresponding to the $D_s^*(B_s^*) - D(B) - K$ vertex:

$$\Pi_\mu^{D(B)} = \frac{\langle 0|\eta^{K}|K(p')\rangle\langle 0|\eta^{D(B)}|D(B)(q)\rangle\langle K(p')D(B)(q)|D_s^*(B_s^*)(p,\epsilon)\rangle\langle D_s^*(B_s^*)(p,\epsilon)|\eta_\mu^{D_s^*(B_s^*)}|0\rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)(p'^2 - m_K^2)} + \ldots, \tag{13}$$

(iii) physical representation corresponding to the $D_s^*(B_s^*) - D(B) - K_1^{a(b)}$ vertex:

$$\begin{aligned}
\Pi_{\mu\nu}^{D(B)} = {}& \frac{\langle 0|\eta^{D(B)}|D(B)(q)\rangle\langle D_s^*(B_s^*)(p,\epsilon)|\eta_\mu^{D_s^*(B_s^*)}|0\rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)}\bigg[\frac{\langle 0|\eta_\nu^{K_1}|K_1^a(p',\epsilon')\rangle\langle K_1^a(p',\epsilon')D(B)(q)|D_s^*(B_s^*)(p,\epsilon)\rangle}{(p'^2 - m_{K_1^a}^2)}\\
&+ \frac{\langle 0|\eta_\nu^{K_1}|K_1^b(p',\epsilon')\rangle\langle K_1^b(p',\epsilon')D(B)(q)|D_s^*(B_s^*)(p,\epsilon)\rangle}{(p'^2 - m_{K_1^b}^2)}\bigg] + \ldots,
\end{aligned} \tag{14}$$

$$\begin{aligned}
\Pi_{\mu\nu\nu'}^{D(B)} = {}& \frac{\langle 0|\eta^{D(B)}|D(B)(q)\rangle\langle D_s^*(B_s^*)(p,\epsilon)|\eta_\mu^{D_s^*(B_s^*)}|0\rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)}\bigg[\frac{\langle 0|\eta_{\nu\nu'}^{K_1}|K_1^a(p',\epsilon')\rangle\langle K_1^a(p',\epsilon')D(B)(q)|D_s^*(B_s^*)(p,\epsilon)\rangle}{(p'^2 - m_{K_1^a}^2)}\\
&+ \frac{\langle 0|\eta_{\nu\nu'}^{K_1}|K_1^b(p',\epsilon')\rangle\langle K_1^b(p',\epsilon')D(B)(q)|D_s^*(B_s^*)(p,\epsilon)\rangle}{(p'^2 - m_{K_1^b}^2)}\bigg] + \ldots,
\end{aligned} \tag{15}$$

where .... represents the contributions of the higher states and continuum, and ϵ and ϵ' are the polarization vectors associated with the D_s^* and $K_1^{a(b)}$ mesons, respectively. From the above equations it is clear that to proceed we need to define the following matrix elements in terms of decay constants as well as strong coupling constants:

$$\begin{aligned}
\langle 0|\eta^{K_0^*}|K_0^*(p')\rangle &= m_{K_0^*} f_{K_0^*},\\
\langle 0|\eta^{K}|K(p')\rangle &= \frac{m_K^2 f_K}{m_u+m_s},\\
\langle 0|\eta^{D(B)}|D(B)(q)\rangle &= \frac{m_{D(B)}^2 f_{D(B)}}{m_{c(b)}+m_u},\\
\langle 0|\eta^{D_s(B_s)}|D_s(B_s)(q)\rangle &= \frac{m_{D_s(B_s)}^2 f_{D_s(B_s)}}{m_{c(b)}+m_s},\\
\langle D_s^*(B_s^*)(p,\epsilon)|\eta_\mu^{D_s^*(B_s^*)}|0\rangle &= m_{D_s^*(B_s^*)} f_{D_s^*(B_s^*)} \epsilon^*{}_\mu,\\
\langle K_0^*(p')D(B)(q)|D_s(B_s)(p)\rangle &= g_{D_sDK_0^*(B_sBK_0^*)}\, p\cdot p',\\
\langle K(p')D(B)(q)|D_s^*(B_s^*)(p,\epsilon)\rangle &= g_{D_s^*DK(B_s^*BK)}(p'-q)\cdot\epsilon,\\
\langle K_1^{a(b)}(p',\epsilon')D(B)(q)|D_s^*(B_s^*)(p,\epsilon)\rangle &= g_{D_s^*DK_1^{a(b)}(B_s^*BK_1^{a(b)})}\{m_{D_s^*(B_s^*)}^2(\epsilon'^*.\epsilon)+(\epsilon'^*.p)(\epsilon.p')\},
\end{aligned} \tag{16}$$

and

$$\left.\begin{aligned}
\langle 0|\eta_\nu^{K_1}|K_1^a(p',\epsilon')\rangle &= m_{K_1^a} f_{K_1^a}\epsilon'_\nu,\\
\langle 0|\eta_{\nu\nu'}^{K_1}|K_1^b(p',\epsilon')\rangle &= f_{K_1^{b\perp}}(\epsilon'_\nu p'_{\nu'}-\epsilon'_{\nu'}p'_\nu)
\end{aligned}\right\} G\text{-parity conserving definitions,}$$

$$\left.\begin{aligned}
\langle 0|\eta_{\nu\nu'}^{K_1}|K_1^a(p',\epsilon')\rangle &= f_{K_1^a} a_0^{\perp,K_1^a}(\epsilon'_\nu p'_{\nu'}-\epsilon'_{\nu'}p'_\nu),\\
\langle 0|\eta_\nu^{K_1}|K_1^b(p',\epsilon')\rangle &= m_{K_1^b} f_{K_1^{b\perp}} a_0^{\parallel,K_1^b}\epsilon'_\nu
\end{aligned}\right\} G\text{-parity violating definitions,} \tag{17}$$

where $f_{K_0^*}$, f_K, $f_{D(B)}$, $f_{D_s(B_s)}$ and $f_{D_s^*(B_s^*)}$ are leptonic decay constants of the K_0^*, K, $D(B)$, $D_s(B_s)$, and $D_s^*(B_s^*)$ mesons, respectively. The $f_{K_1^a}$ and $f_{K_1^{b\perp}}$ are decay constants encountered to the calculations from both definitions of the G-parity conserving and violating matrix elements for the axial K_1^a and K_1^b states (for more details see [15,20,40,41]). The $a_0^{\perp,K_1^a}$ and $a_0^{\parallel,K_1^b}$ are zeroth order Gegenbauer moments. In the above equations, the $g_{D_sDK_0^*(B_sBK_0^*)}$, $g_{D_s^*DK(B_s^*BK)}$, and $g_{D_s^*DK_1^{a(b)}(B_s^*BK_1^{a(b)})}$ are strong coupling constants, which we are going to obtain QCD sum rules for them in this section.

Using Eqs. (12)–(16), and summing over the polarization vectors using the

$$\epsilon'_\nu\epsilon'^*_\theta = -g_{\nu\theta}+\frac{p'_\nu p'_\theta}{m_{K_1^{a(b)}}^2}, \qquad \epsilon_\eta\epsilon^*_\mu = -g_{\eta\mu}+\frac{p_\eta p_\mu}{m_{D_s^*(B_s^*)}^2}, \tag{18}$$

the final physical representations of the correlation functions for each vertices in the case of $D(B)$ off shell is obtained as

(i) $D_s(B_s) - D(B) - K_0^*$ vertex:

$$\Pi^{D(B)} = g^{D(B)}_{D_sDK_0^*(B_sBK_0^*)}(q^2) \times \frac{f_{K_0^*} m_{K_0^*} \frac{f_{D(B)}m_{D(B)}^2}{m_{c(b)}+m_u}\frac{f_{D_s(B_s)}m_{D_s(B_s)}^2}{m_{c(b)}+m_s}}{2(q^2-m_{D(B)}^2)(p'^2-m_{K_0^*}^2)(p^2-m_{D_s(B_s)}^2)} \times (m_{D_s(B_s)}^2+m_{K_0^*}^2-q^2)+\ldots, \tag{19}$$

(ii) $D_s^*(B_s^*) - D(B) - K$ vertex:

$$\Pi_\mu^{D(B)} = g^{D(B)}_{D_s^*DK(B_s^*BK)}(q^2) \times \frac{f_{D_s^*(B_s^*)} m_{D_s^*(B_s^*)} \frac{f_K m_K^2}{m_s+m_u}\frac{f_{D(B)}m_{D(B)}^2}{m_{c(b)}+m_u}}{(q^2-m_{D(B)}^2)(p'^2-m_K^2)(p^2-m_{D_s^*(B_s^*)}^2)} \times\left[\left(1+\frac{m_K^2-q^2}{m_{D_s^*}^2}\right)p_\mu-2p'_\mu\right]+\ldots, \tag{20}$$

(iii) $D_s^*(B_s^*) - D(B) - K_1$ vertex:

$$\Pi^{D(B)}_{\mu\nu} = \Big(g^{D(B)}_{D_s^* D K_1^a (B_s^* B K_1^a)}(q^2) \frac{m_{K_1^a} f_{K_1^a}}{(p'^2 - m^2_{K_1^a})} + g^{D(B)}_{D_s^* D K_1^b (B_s^* B K_1^b)}(q^2) \frac{m_{K_1^b} f_{K_1^{b\perp}} a_0^{\parallel, K_1^b}}{(p'^2 - m^2_{K_1^b})}\Big) \times \frac{\frac{f_{D(B)} m^2_{D(B)}}{m_{c(b)}+m_u} f_{D_s^*(B_s^*)} m_{D_s^*(B_s^*)}}{(q^2 - m^2_{D(B)})(p^2 - m^2_{D_s^*(B_s^*)})} \times \{m^2_{D_s^*(B_s^*)} g_{\mu\nu} + \text{other structures}\} + \ldots, \tag{21}$$

$$\Pi^{D(B)}_{\mu\nu\nu'} = \Big(g^{D(B)}_{D_s^* D K_1^a (B_s^* B K_1^a)}(q^2) \frac{f_{K_1^a} a_0^{\perp, K_1^a}}{(p'^2 - m^2_{K_1^a})} + g^{D(B)}_{D_s^* D K_1^b (B_s^* B K_1^b)}(q^2) \frac{f_{K_1^{b\perp}}}{(p'^2 - m^2_{K_1^b})}\Big) \times \frac{\frac{f_{D(B)} m^2_{D(B)}}{m_{c(b)}+m_u} f_{D_s^*(B_s^*)} m_{D_s^*(B_s^*)}}{(q^2 - m^2_{D(B)})(p^2 - m^2_{D_s^*(B_s^*)})} \times \{m^2_{D_s^*(B_s^*)}(g_{\mu\nu} p'_{\nu'} - g_{\mu\nu'} p'_{\nu}) + \text{other structures}\} + \ldots, \tag{22}$$

where to calculate the coupling constants, we will choose the structures p_μ and $g_{\mu\nu}(g_{\mu\nu} p'_{\nu'} - g_{\mu\nu'} p'_{\nu})$ from both sides of the correlation functions corresponding to the vertices containing the K and K_1 with current $\eta_\nu^{K_1}$ (K_1 with current $\eta_{\nu\nu'}^{K_1}$), respectively. In a similar way, one can easily obtain the physical representations of the correlation functions associated with the $K_0^*(K)(K_1)$ off shell.

Now, we calculate the QCD or theoretical sides of the considered correlation functions. The QCD representations of the correlation functions are obtained in the deep Euclidean region, where $p^2 \to -\infty$ and $p'^2 \to -\infty$ via operator product expansion. To this aim, each correlation function in QCD side is written in terms of the perturbative and nonperturbative parts as

$$\Pi^i = \Pi^i_{\text{per}} + \Pi^i_{\text{nonper}}, \tag{23}$$

where i stands for $D(B)$ or $K_0^*(K)(K_1)$ and the perturbative parts are defined in terms of a double dispersion integral as follows:

$$\Pi^i_{\text{per}} = -\frac{1}{4\pi^2}\int ds' \int ds \frac{\rho^i(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \tag{24}$$

where $\rho^i(s, s', q^2)$ are called spectral densities. In order to obtain the spectral density, we need to calculate the bare loop diagrams (a) and (d) in Fig. 1 for $D(B)$ and $K_0^*(K)(K_1)$ off shell, respectively. We calculate these diagrams in

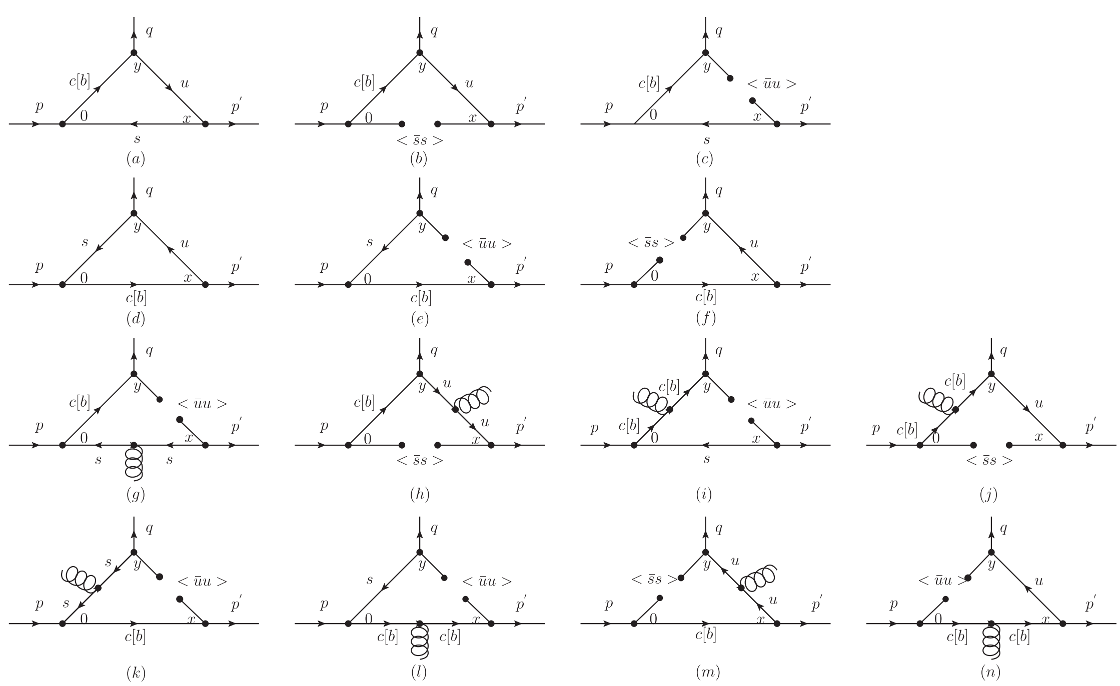

FIG. 1. Diagrams considered in the calculations. The first and third line diagrams refer to the $B(D)$ off shell, and the second and fourth line diagrams show the case when $K_0^*(K)(K_1)$ is off shell.

terms of the usual Feynman integrals with the help of the Cutkosky rules, where the quark propagators are replaced by Dirac delta function, i.e., $\frac{1}{q^2-m^2} \rightarrow (-2\pi i)\delta(q^2 - m^2)$. As a result, the spectral densities are obtained as follows:

(i) $D_s(B_s) - D(B) - K_0^*$ vertex:

(i) $D(B)$ off shell:

$$\rho^{D(B)}(s, s', q^2) = \frac{N_c}{2\lambda^{1/2}(s, s', q^2)}\{m_s(m_u(m_s + m_u) - q^2) - sm_u - m_{c(b)}((m_s + m_u)^2 - s' - m_{c(b)}(m_s + m_u))\}, \tag{25}$$

(ii) K_0^* off shell:

$$\rho^{K_0^*}(s, s', q^2) = \frac{N_c}{2\lambda^{1/2}(s, s', q^2)}\{m_s s' + m_{c(b)}((m_s + m_u)^2 - q^2) - m_{c(b)}^2(m_s + m_u) + m_u(s - m_s(m_s + m_u))\}, \tag{26}$$

(ii) $D_s^*(B_s^*) - D(B) - K$ vertex:

(i) $D(B)$ off shell:

$$\begin{aligned}\rho^{D(B)}(s, s', q^2) = \frac{N_c}{\lambda^{3/2}(s, s', q^2)}&[(m_u - m_s)(q^2 - s)(m_{c(b)}m_s^2 + m_u(s - m_s^2 - q^2)) - s'(-m_s^3 m_u + 2m_{c(b)}^3(m_u - m_s)\\ &- 2m_s^2 q^2 + m_{c(b)}^2(2m_s m_u + q^2 - s) + q^2(s - q^2) + m_s m_u(s + q^2) + m_{c(b)}(m_s - m_u)(m_s^2 + q^2 + s))\\ &- s'^2(m_{c(b)}^2 - m_{c(b)}m_s + m_{c(b)}m_u + q^2)],\end{aligned} \tag{27}$$

(ii) K off shell:

$$\begin{aligned}\rho^{K}(s, s', q^2) = \frac{N_c}{\lambda^{3/2}(s, s', q^2)}&[(m_{c(b)} - m_u)(q^2 - s)(m_{c(b)}^2(m_{c(b)} - m_u) + m_u(-m_s m_u - q^2 + s)) + (m_{c(b)}^3(m_s - m_u)\\ &+ 2m_s^3 m_u + m_{c(b)}^2(-m_s m_u - 2q^2) + m_s^2(q^2 - s) + q^2(s - q^2) - m_s m_u(q^2 + s)\\ &+ m_{c(b)}(-2m_s^3 + 2m_s^2 m_u + m_u(q^2 + s) + m_s(q^2 + s)))s' + (-m_{c(b)}m_s + m_s^2 + m_s m_u + q^2)s'^2],\end{aligned} \tag{28}$$

(iii) $D_s^*(B_s^*) - D(B) - K_1$ vertex:

(i) $D(B)$ off shell:

$$\begin{aligned}\rho_1^{D(B)}(s, s', q^2) = -2N_c I_0(s, s', q^2)&[2m_s^3 + 2m_s^2 m_u - 2m_{c(b)}m_s(m_s + m_u) + m_s(s + s' - q^2) + 4A(m_{c(b)} - m_u)\\ &+ B(2m_u s + m_{c(b)}(q^2 - s - s') + m_s(-q^2 + 3s + s')) + C(-2m_{c(b)}s' + m_u(-q^2 + s + s')\\ &+ m_s(-q^2 + s + 3s'))],\end{aligned} \tag{29}$$

$$\begin{aligned}\rho_2^{D(B)}(s, s', q^2) = 4N_c I_0(s, s', q^2)&[2A + (m_{c(b)}m_s - m_s^2) - (B + 2G)s + H(q^2 - s - s') + C(m_{c(b)}(m_s + m_u)\\ &- m_s(m_s + m_u) + q^2 - s - s')],\end{aligned} \tag{30}$$

(ii) K_1 off shell:

$$\begin{aligned}\rho_1^{K_1}(s, s', q^2) = 2N_c I_0(s, s', q^2)&[2m_{c(b)}(m_{c(b)}^2 + m_s m_u - m_{c(b)}(m_s + m_u) + s') - 4D(m_{c(b)} - m_u)\\ &+ E(2m_{c(b)} - m_s - m_u)(q^2 - s - s') + 2Fs'(-2m_{c(b)} + m_s + m_u)],\end{aligned} \tag{31}$$

$$\rho_2^{K_1}(s, s', q^2) = 4N_c I_0(s, s', q^2)[-2D + m_{c(b)}^2 - m_{c(b)}m_s - Es - F(m_{c(b)}(m_{c(b)} - m_s - m_u) + m_s m_u)], \tag{32}$$

where ρ_1 and ρ_2 correspond to the currents $\eta_\nu^{K_1}$ and $\eta_{\nu\nu'}^{K_1}$, respectively, and

$$
\begin{aligned}
A &= \frac{1}{2\Delta}\{m_s^4 q^2 + m_{c(b)}^4 s' + q^2 s s' + m_{c(b)}^2 (m_s^2(-q^2+s-s') + s'(-q^2-s+s')) - m_s^2 q^2(-q^2+s+s')\},\\
B &= \frac{1}{\Delta}\{m_s^2(q^2-s+s') - s'(2m_{c(b)}^2 - q^2 - s + s')\},\\
C &= \frac{1}{\Delta}\{m_{c(b)}^2(-q^2+s+s') + s(q^2-s+s') - m_s^2(-q^2-s+s')\},\\
D &= \frac{1}{2\Delta}\{m_{c(b)}^4 q^2 + m_s^4 s' + q^2 s s' - m_{c(b)}^2 q^2(-q^2+s+s') + m_s^2(m_{c(b)}^2(-q^2+s-s') + s'(-q^2-s+s'))\},\\
E &= \frac{1}{\Delta}\{s'(2m_s^2 - q^2 - s + s') - m_{c(b)}^2(q^2-s+s')\},\\
F &= \frac{1}{\lambda(s,s',q^2)}\{(-m_s^2 + m_{c(b)}^2 + s)(-q^2+s+s') - 2s(m_{c(b)}^2 + s')\},\\
G &= \frac{1}{\Delta^2}\{3m_{c(b)}^4 s'(q^2-s-s') + m_s^4(2q^4 - (s-s')^2 - q^2(s+s')) - ss'(-2q^4 + (s-s')^2 + q^2(s+s'))\\
&\quad + m_s^2(q^6 - q^4(s+s') + (s-s')^2(s+s') - q^2(s^2 - 6ss' + s'^2)) - 2m_{c(b)}^2(s'(q^4 - 2s^2\\
&\quad + q^2(s-2s') + ss' + s'^2) + m_s^2(q^4 + s^2 + ss' - 2s'^2 + q^2(-2s+s')))\},\\
H &= \frac{1}{\Delta^2}\{s^2(q^4 - 2q^2(s-2s') + (s-s')^2) + m_s^4(q^4 + 2q^2(2s-s') + (s-s')^2)\\
&\quad + m_{c(b)}^4(q^4 + s^2 + 4ss' + s'^2 - 2q^2(s+s')) - 2m_s^2 s(-2q^4 + (s-s')^2 + q^2(s+s'))\\
&\quad - 2m_{c(b)}^2(m_s^2(q^4 - 2s^2 + q^2(s-s') + ss' + s'^2) + s(q^4 + s^2 + ss' - 2s'^2 + q^2(-2s+s')))\},
\end{aligned} \tag{33}
$$

and

$$
I_0(s,s',q^2) = \frac{1}{4\lambda^{1/2}(s,s',q^2)} \qquad \Delta = q^4 + (s-s')^2 - 2q^2(s+s') \qquad \lambda(a,b,c) = a^2 + b^2 + c^2 - 2ac - 2bc - 2ab. \tag{34}
$$

In the spectral densities, $N_c = 3$ is the color number, and we have kept terms linear in m_u.

Now, we proceed to calculate the nonperturbative contributions in QCD side. We consider the quark-quark and quark-gluon condensate diagrams presented as (b), (c), (e), (f), (g), (h), (i), (j), (k), (l), (m), and (n) in Fig. 1. Contributions of the diagrams (c), (e), (f), (g), (i), (k), (l), (m), and (n) in Fig. 1 are zero since applying double Borel transformation with respect to the both variables p^2 and p'^2 will kill their contributions because of only one variable appearing in the denominator in these cases. Hence, we consider contributions of only diagrams (b), (h), and (j) in Fig. 1. As a result, we obtain:

(i) $D_s(B_s) - D(B) - K_0^*$ vertex:

(i) $D(B)$ off shell:

$$
\Pi_{\text{nonper}}^{D(B)} = \frac{\langle \bar{s}s\rangle}{2}\left\{\frac{2m_{c(b)}m_u - m_{c(b)}^2 + q^2}{rr'} - \frac{1}{r} - \frac{1}{r'} + \frac{m_0^2(4m_{c(b)}m_u - m_{c(b)}^2 + q^2)}{4r^2 r'} - \frac{m_0^2}{4rr'} - \frac{m_0^2}{4r^2} + \frac{m_0^2(m_{c(b)}^2 - 4m_{c(b)}m_u - q^2)}{4rr'^2} + \frac{m_0^2}{4r^2} + \frac{m_0^2}{4rr'}\right\}, \tag{35}
$$

(ii) K_0^* off shell:

$$
\Pi_{\text{nonper}}^{K_0^*} = 0, \tag{36}
$$

(ii) $D_s^*(B_s^*) - D(B) - K$ vertex:

(i) $D(B)$ off shell:

$$
\Pi_{\text{nonper}}^{D(B)} = -\langle \bar{s}s\rangle\left\{\frac{m_u}{rr'} + \frac{m_0^2 m_u}{4r^2 r'} + \frac{m_0^2 m_u}{2rr'^2}\right\}, \tag{37}
$$

(ii) K off shell:

$$\Pi^{K}_{\rm nonper} = 0, \tag{38}$$

(iii) $D^*_s(B^*_s) - D(B) - K_1$ vertex:
(i) $D(B)$ off shell:

$$\Pi^{D(B)}_{\rm nonper1} = -\langle \bar{s}s\rangle \left[\frac{m_0^2}{24}\left(\frac{m^2_{c(b)} - q^2}{r^2 r'} + \frac{2}{rr'} + \frac{1}{r^2} + \frac{1}{r'^2} + \frac{m^2_{c(b)} - q^2}{rr'^2}\right) + \frac{q^2 + 2m_u m_{c(b)} - m^2_{c(b)}}{2rr'} - \frac{1}{2r} - \frac{1}{r'}\right], \tag{39}$$

$$\Pi^{D(B)}_{\rm nonper2} = -\langle \bar{s}s\rangle \left[\frac{m_{c(b)}}{rr'} - \frac{m_0^2 m_{c(b)}}{24rr'^2}\right], \tag{40}$$

(ii) K_1 off shell:

$$\Pi^{K_1}_{\rm nonper1} = 0, \tag{41}$$

$$\Pi^{K_1}_{\rm nonper2} = 0, \tag{42}$$

where $r = p^2 - m^2_{c(b)}$ and $r' = p'^2 - m_u^2$. The $\Pi_{\rm nonper1}$ and $\Pi_{\rm nonper2}$ correspond to also the currents $\eta^{K_1}_{\nu}$ and $\eta^{K_1}_{\nu\nu'}$, respectively. In this step, we equate the physical side in the case of K_0^* and the coefficients of the selected structures in physical sides of K and K_1 to the corresponding QCD sides and obtain QCD sum rules for the considered strong coupling constants. To suppress the contributions of the higher states and continuum, we also apply the double Borel transformation with respect to the variables $p^2(p^2 \to M^2)$ and $p'^2(p'^2 \to M'^2)$. Finally, we get the following sum rules for the considered coupling constants:
(i) $D_s(B_s) - D(B) - K_0^*$ vertex:
(i) $D(B)$ off shell:

$$g^{D(B)}_{D_s D K_0^*(B_s B K_0^*)}(q^2) = \frac{2(q^2 - m^2_{D(B)})(m_{c(b)} + m_u)(m_{c(b)} + m_s)}{m^2_{D_s(B_s)} m^2_{D(B)} m_{K_0^*} f_{D_s(B_s)} f_{D(B)} f_{K_0^*}(m^2_{D_s(B_s)} + m^2_{K_0^*} - q^2)} e^{m^2_{D_s(B_s)}/M^2} e^{m^2_{K_0^*}/M'^2} \times\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_s+m_u)^2}^{s_0'} ds' \rho^{D(B)}(s,s',q^2)\theta[1-(f^{D(B)}(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2} + \hat{B}\Pi^{D(B)}_{\rm nonper}\right], \tag{43}$$

(ii) K_0^* off shell:

$$g^{K_0^*}_{D_s D K_0^*(B_s B K_0^*)}(q^2) = \frac{-2(q^2 - m^2_{K_0^*})(m_{c(b)} + m_u)(m_{c(b)} + m_s)}{m^2_{D_s(B_s)} m^2_{D(B)} m_{K_0^*} f_{D_s(B_s)} f_{D(B)} f_{K_0^*}(m^2_{D_s(B_s)} + m^2_{D(B)} - q^2)} e^{m^2_{D_s(B_s)}/M^2} e^{m^2_{D_s(B_s)}/M'^2} \times\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_{c(b)}+m_u)^2}^{s_0'} ds' \rho^{K_0^*}(s,s',q^2)\theta[1-(f^{K_0^*}(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2}\right], \tag{44}$$

(ii) $D^*_s(B^*_s) - D(B) - K$ vertex:
(i) $D(B)$ off shell:

$$g^{D(B)}_{D^*_s D K(B^*_s B K)}(q^2) = \frac{(q^2 - m^2_{D(B)})(m_{c(b)} + m_u)(m_s + m_u)}{f_{D^*_s(B^*_s)} f_{D(B)} f_K m_{D^*_s(B^*_s)} m_K^2 m^2_{D(B)}(1 + \frac{m_K^2 - q^2}{m^2_{D^*_s(B^*_s)}})} e^{m^2_{D^*_s(B^*_s)}/M^2} e^{m_K^2/M'^2} \times\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_s+m_u)^2}^{s_0'} ds' \rho^{D(B)}(s,s',q^2)\theta[1-(f^{D(B)}(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2} + \hat{B}\Pi^{D(B)}_{\rm nonper}\right] \tag{45}$$

(ii) K off shell:

$$g^{K}_{D_s^*DK(B_s^*BK)}(q^2)=\frac{(q^2-m_K^2)(m_{c(b)}+m_u)(m_s+m_u)}{f_{D_s^*(B_s^*)}f_{D(B)}f_K m_{D_s^*(B_s^*)}m_K^2 m_{D(B)}^2(1+\frac{m_{D(B)}^2-q^2}{m^2_{D_s^*(B_s^*)}})}e^{m^2_{D_s^*(B_s^*)}/M^2}e^{m^2_{D(B)}/M'^2}\times\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0}ds\int_{(m_{c(b)}+m_u)^2}^{s'_0}ds'\rho^K(s,s',q^2)\theta[1-(f^K(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2}\right], \tag{46}$$

(iii) $D_s^*(B_s^*)-D(B)-K_1$ vertex:
(i) $D(B)$ off shell:

$$\left(g^{D(B)}_{D_s^*DK_1^a(B_s^*BK_1^a)}(q^2)m_{K_1^a}f_{K_1^a}e^{-m^2_{K_1^a}/M'^2}+g^{D(B)}_{D_s^*DK_1^b(B_s^*BK_1^b)}(q^2)m_{K_1^b}f_{K_1^{b\perp}}a_0^{\parallel,K_1^b}e^{-m^2_{K_1^b}/M'^2}\right)=\frac{(q^2-m^2_{D(B)})}{f_{D_s^*(B_s^*)}f_{D(B)}\frac{m^2_{D(B)}}{m_{c(b)}+m_u}m^3_{D_s^*(B_s^*)}}e^{m^2_{D_s^*(B_s^*)}/M^2}\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0}ds\int_{(m_s+m_u)^2}^{s'_0}ds'\rho_1^{D(B)}(s,s',q^2)\times\theta[1-(f^{D(B)}(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2}+\hat{B}\Pi^{D(B)}_{\text{nonper1}}\right], \tag{47}$$

$$\left(g^{D(B)}_{D_s^*DK_1^a(B_s^*BK_1^a)}(q^2)f_{K_1^a}a_0^{\perp,K_1^a}e^{-m^2_{K_1^a}/M'^2}+g^{D(B)}_{D_s^*DK_1^b(B_s^*BK_1^b)}(q^2)f_{K_1^{b\perp}}e^{-m^2_{K_1^b}/M'^2}\right)=\frac{(q^2-m^2_{D(B)})}{f_{D_s^*(B_s^*)}f_{D(B)}\frac{m^2_{D(B)}}{m_{c(b)}+m_u}m^3_{D_s^*(B_s^*)}}e^{m^2_{D_s^*(B_s^*)}/M^2}\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0}ds\int_{(m_s+m_u)^2}^{s'_0}ds'\rho_2^{D(B)}(s,s',q^2)\times\theta[1-(f^{D(B)}(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2}+\hat{B}\Pi^{D(B)}_{\text{nonper2}}\right], \tag{48}$$

(ii) K_1 off shell:

$$\left(g^{K_1^a}_{D_s^*DK_1^a(B_s^*BK_1^a)}(q^2)\frac{m_{K_1^a}f_{K_1^a}}{(q^2-m^2_{K_1^a})}+g^{K_1^b}_{D_s^*DK_1^b(B_s^*BK_1^b)}(q^2)\frac{m_{K_1^b}f_{K_1^{b\perp}}a_0^{\parallel,K_1^b}}{(q^2-m^2_{K_1^b})}\right)=\frac{1}{f_{D_s^*(B_s^*)}f_{D(B)}\frac{m^2_{D(B)}}{m_{c(b)}+m_u}m^3_{D_s^*(B_s^*)}}e^{m^2_{D_s^*(B_s^*)}/M^2}e^{m^2_{D(B)}/M'^2}\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0}ds\int_{(m_{c(b)}+m_u)^2}^{s'_0}ds'\rho_1^{K_1}(s,s',q^2)\times\theta[1-(f^{K_1}(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2}+\hat{B}\Pi^{K_1}_{\text{nonper1}}\right], \tag{49}$$

$$\left(g^{K_1^a}_{D_s^*DK_1^a(B_s^*BK_1^a)}(q^2)\frac{f_{K_1^a}a_0^{\perp,K_1^a}}{(q^2-m^2_{K_1^a})}+g^{K_1^b}_{D_s^*DK_1^b(B_s^*BK_1^b)}(q^2)\frac{f_{K_1^{b\perp}}}{(q^2-m^2_{K_1^b})}\right)=\frac{1}{f_{D_s^*(B_s^*)}f_{D(B)}\frac{m^2_{D(B)}}{m_{c(b)}+m_u}m^3_{D_s^*(B_s^*)}}e^{m^2_{D_s^*(B_s^*)}/M^2}e^{m^2_{D(B)}/M'^2}\left[-\frac{1}{4\pi^2}\int_{(m_{c(b)}+m_s)^2}^{s_0}ds\int_{(m_{c(b)}+m_u)^2}^{s'_0}ds'\rho_2^{K_1}(s,s',q^2)\times\theta[1-(f^{K_1}(s,s'))^2]e^{-s/M^2}e^{-s'/M'^2}+\hat{B}\Pi^{K_1}_{\text{nonper2}}\right], \tag{50}$$

where the $\hat{B}\Pi_{\text{nonper}}$ represents the double Borel transformation of the nonperturbative part in each case, s_0 and s'_0 are the continuum thresholds and the functions $f^i(s,s')$ inside the step functions are determined requiring that the arguments of the three δ functions coming from the Cutkosky rule vanish simultaneously. As a result, we find:
(i) $D(B)$ off shell:

$$f^{D(B)}(s,s') = \frac{2s(m_s^2 - m_u^2 + s') + (m_{c(b)}^2 - m_s^2 - s)(-q^2 + s + s')}{\lambda^{1/2}(m_{c(b)}^2, m_s^2, s)\lambda^{1/2}(s, s', q^2)}, \tag{51}$$

(ii) $K_0^*(K)(K_1)$ off shell:

$$f_1^{K_0^*(K)(K_1)}(s,s') = \frac{2s(-m_{c(b)}^2 + m_u^2 - s') + (m_{c(b)}^2 - m_s^2 + s)(-q^2 + s + s')}{\lambda^{1/2}(m_{c(b)}^2, m_s^2, s)\lambda^{1/2}(s, s', q^2)}. \tag{52}$$

Here, we should stress that the physical regions are imposed by the limits on the integrals and step functions in the integrands in the sum rules expressions. In order to subtract the contributions of the higher states and continuum, the quark-hadron duality assumption in the following form is used:

$$\rho^{\text{higherstates}}(s,s') = \rho^{\text{OPE}}(s,s')\theta(s - s_0)\theta(s' - s_0'). \tag{53}$$

The double Borel transformation used in calculations is also defined in the following way:

$$\hat{B}\frac{1}{(p^2 - m_1^2)^m}\frac{1}{(p'^2 - m_2^2)^n} \to (-1)^{m+n}\frac{1}{\Gamma(m)}\frac{1}{\Gamma(n)}e^{-m_1^2/M^2}e^{-m_2^2/M'^2}\frac{1}{(M^2)^{m-1}(M'^2)^{n-1}}. \tag{54}$$

At the end of this section, we would like to mention that using Eqs. (1) and (16) the couplings to $K_1(1270)$ and $K_1(1400)$ are obtained in terms of the couplings to the $K_1^{a(b)}$ as

$$\begin{aligned} g_{D_s^*DK_1(1270)(B_s^*BK_1(1270))} &= g_{D_s^*DK_1^a(B_s^*BK_1^a)}\sin\theta + g_{D_s^*DK_1^b(B_s^*BK_1^b)}\cos\theta \\ g_{D_s^*DK_1(1400)(B_s^*BK_1(1400))} &= g_{D_s^*DK_1^a(B_s^*BK_1^a)}\cos\theta - g_{D_s^*DK_1^b(B_s^*BK_1^b)}\sin\theta \end{aligned} \tag{55}$$

III. NUMERICAL ANALYSIS

In the present section, we numerically analyze the expressions of QCD sum rules obtained for the considered strong coupling constants. Some input parameters used in the calculations are $m_K = (493.677 \pm 0.016)$ MeV, $m_{K_0^*}(800) = (672 \pm 40)$ MeV, $m_{K_0^*}(1430) = (1425 \pm 50)$ MeV, $m_{K_1}(1270) = (1272 \pm 7)$ MeV, $m_{K_1}(1400) = (1403 \pm 7)$ MeV, $m_D = (1.8648 \pm 0.000\,14)$ GeV, $m_B = (5.2792 \pm 0.0003)$ GeV, $m_{D_s} = (1.968\,47 \pm 0.000\,33)$ MeV, $m_{B_s} = (5.3663 \pm 0.0006)$ MeV, $m_{D_s^*} = (2.1123 \pm 0.0005)$ GeV, $m_{B_s^*} = (5.4154 \pm 0.0014)$ GeV [42], $m_c = 1.3$ GeV, $m_b = 4.7$ GeV, $m_s = 0.14$ GeV [43], $f_K = 160$ MeV [44], $f_{K_0^*(800)}(1\text{ GeV}) = (340 \pm 20)$ MeV, $f_{K_0^*(1430)}(1\text{ GeV}) = (445 \pm 50)$ MeV [45], $f_{D_s^*} = (272 \pm 16^{0}_{-20})$ MeV, $f_{B_s^*} = (229 \pm 20^{31}_{-16})$ MeV [46], $f_B = 190 \pm 13$ MeV [47], $f_D = (202 \pm 41 \pm 17)$ MeV [48], $f_{D_s} = (286 \pm 44 \pm 41)$ MeV [49], $f_{B_s} = 196$ MeV [50], $\langle \bar{s}s\rangle = 0.8\langle \bar{u}u\rangle = -0.8(0.24 \pm 0.01)^3$ GeV^3 [43], $m_0^2 = (0.8 \pm 0.2)$ GeV^2 [51], $m_{K_1^a} = 1.31$ GeV, $m_{K_1^b} = 1.34$ GeV, $f_{K_1^a}(1\text{ GeV}) = 0.25$ GeV, $f_{K_1^b\perp}(1\text{ GeV}) = 0.19$ GeV, $a_0^{\parallel,K_1^b}(1\text{ GeV}) = -0.19 \pm 0.07$ and $a_0^{\perp,K_1^A}(1\text{ GeV}) = 0.27^{+0.03}_{-0.17}$ [15,20,40].

The sum rule for strong coupling constants also contains four auxiliary parameters, namely, the continuum thresholds s_0 and s_0' related to the initial and final channels, respectively, as well as Borel mass parameters M^2 and M'^2. These quantities are mathematical objects, so according to the general criteria and standard procedure in QCD sum rules, our physical results should be insensitive to them. Therefore, we shall look for working regions of these quantities at which the dependence of coupling constants on these auxiliary parameters are weak. The working regions for the Borel mass parameters M^2 and M'^2 are determined demanding that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from the higher dimensional operators are small. Our calculations lead to the following working regions common for all cases:

(i) $D_s^{(*)}DK_0^*(K)(K_1)$ vertex:

(i) D off shell: 8 GeV$^2 \le M^2 \le 25$ GeV^2 and 5 GeV$^2 \le M'^2 \le 15$ GeV^2,

(ii) $K_0^*(K)(K_1)$ off shell: 6 GeV$^2 \le M^2 \le 15$ GeV^2 and 4 GeV$^2 \le M'^2 \le 12$ GeV^2,

(ii) $B_s^{(*)}BK_0^*(K)(K_1)$ vertex:

(i) B off shell: 14 GeV$^2 \le M^2 \le 30$ GeV^2 and 5 GeV$^2 \le M'^2 \le 20$ GeV^2,

(ii) $K_0^*(K)K_1$ off shell: 6 GeV$^2 \le M^2 \le 20$ GeV^2 and 5 GeV$^2 \le M'^2 \le 15$ GeV^2.

The continuum thresholds, s_0 and s_0' are not completely arbitrary but they are correlated to the energy of the first excited states with the same quantum numbers as the considered interpolating currents. Our numerical calculations show that in the regions $(m_i + 0.3)^2 \le s_0 \le (m_i + 0.7)^2$

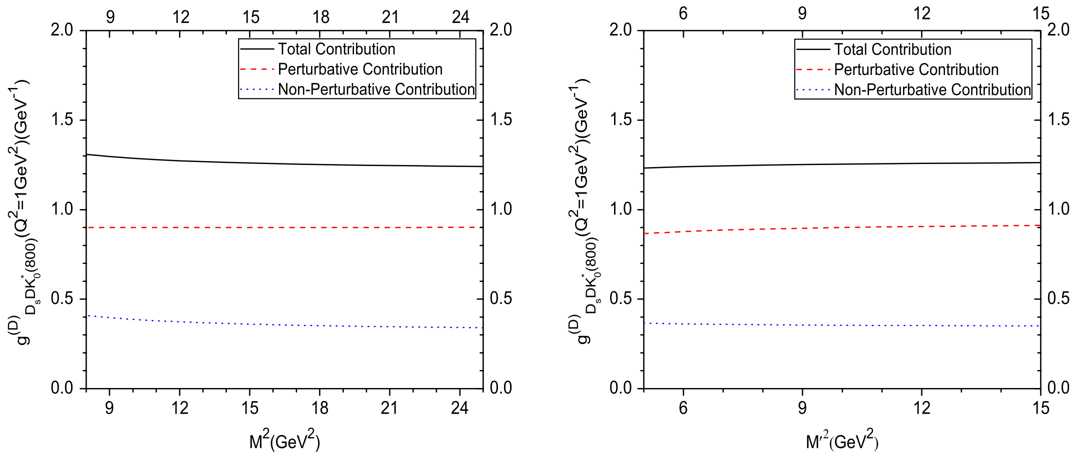

FIG. 2 (color online). *Left:* $g^{(D)}_{D_sDK_0^*(800)}(Q^2 = 1\ \mathrm{GeV}^2)$ as a function of the Borel mass M^2 with $M'^2 = 10\ \mathrm{GeV}^2$. *Right:* $g^{(D)}_{D_sDK_0^*(800)}(Q^2 = 1\ \mathrm{GeV}^2)$ as a function of the Borel mass M'^2 with $M^2 = 17\ \mathrm{GeV}^2$. The continuum thresholds $s_0 = 6.09\ \mathrm{GeV}^2$ and $s'_0 = 1.37\ \mathrm{GeV}^2$ have been used.

and $(m_f + 0.3)^2 \leq s'_0 \leq (m_f + 0.7)^2$, respectively, for the continuum thresholds s and s', our results have weak dependence on these parameters. Here, m_i is the mass of initial particle and the m_f stands for the mass of the final on-shell state. For instance consider the $g^D_{D_sDK_0^*}$ coupling constant at which the D meson is off shell. This coupling constant describe the strong transition $D_s \to DK_0^*$, and for this case $m_i = m_{D_s}$ and $m_f = m_{K_0^*}$.

As an example, we present the dependence of strong coupling constant $g^{(D)}_{D_sDK_0^*(800)}$ on Borel mass parameters at $Q^2 = 1\ \mathrm{GeV}^2$, where $Q^2 = -q^2$ in Fig. 2. This figure demonstrates a good stability of the results with respect to the variations of Borel mass parameters in their working regions.

Now, we proceed to find the Q^2 behavior of the considered strong coupling constants using the working regions for auxiliary parameters. First, we consider the scalar kaon case for both $K_0^*(800)$ and $K_0^*(1430)$. The strong coupling constant in this case obeys from the following Boltzmann function:

$$g(Q^2) = A_1 + \frac{A_2}{1 + \exp[\frac{Q^2 - x_0}{\Delta x}]}\ [\mathrm{GeV}^{-1}]. \qquad (56)$$

The values of the parameters A_1, A_2, x_0 and Δx for the considered coupling constant form factors are given in Table I. The coupling constants are defined as the values of the form factors at $Q^2 = -m^2_{\mathrm{meson}}$, where m_{meson} is the mass of off-shell meson. The results of the coupling constants obtained using $Q^2 = -m^2_{\mathrm{meson}}$ are given in Tables II and III. The final result for each coupling constant is obtained taking the average of the coupling constants obtained from two different off-shell cases. The errors in the numerical results are due to the uncertainties in determination of the working regions for the auxiliary parameters as well as the errors in the input parameters.

In the case of pseudoscalar kaon and D off shell, the strong coupling constant is well described by the following monopolar fit parametrization:

$$g^{(D)}_{D_s^*DK}(Q^2) = \frac{8.76\ (\mathrm{GeV}^2)}{Q^2 + 7.12\ (\mathrm{GeV}^2)}, \qquad (57)$$

The value of coupling constant obtained at $Q^2 = -m^2_{\mathrm{meson}}$ is presented in Table IV.

The result for strong coupling constant of pseudoscalar case and an off-shell K meson can be well fitted by the exponential parametrization

$$g^{(K)}_{D_s^*DK}(Q^2) = 3.55e^{-(Q^2/7.25\ (\mathrm{GeV}^2))} - 0.88, \qquad (58)$$

where using $Q^2 = -m_K^2$, we obtain the result as also presented in Table IV. We also depict the final result for this case taking the average of two above obtained values.

TABLE I. Parameters appearing in the fit function of the coupling constants for the $D_sDK_0^*(800)$, $D_sDK_0^*(1430)$, $B_sBK_0^*(800)$, and $B_sBK_0^*(1430)$ vertices. A_1 and A_2 are in GeV^{-1} units, while x_0 and Δx are in the units of GeV^2.

	A_1	A_2	x_0	Δx
$g^{(D)}_{D_sDK_0^*(800)}(Q^2)$	3.468	−2.741	8.067	4.995
$g^{(K_0^*(800))}_{D_sDK_0^*(800)}(Q^2)$	−0.024	0.772	5.723	1.257
$g^{(D)}_{D_sDK_0^*(1430)}(Q^2)$	4.712	−3.818	24.863	10.985
$g^{(K_0^*(1430))}_{D_sDK_0^*(1430)}(Q^2)$	−0.022	0.772	4.729	1.637
$g^{(B)}_{B_sBK_0^*(800)}(Q^2)$	4.151	−1.932	13.842	12.149
$g^{(K_0^*(800))}_{B_sBK_0^*(800)}(Q^2)$	−0.017	0.547	5.431	1.121
$g^{(B)}_{B_sBK_0^*(1430)}(Q^2)$	2.055	−0.207	11.239	5.084
$g^{(K_0^*(1430))}_{B_sBK_0^*(1430)}(Q^2)$	−0.004	0.255	4.819	1.146

TABLE II. Value of the $g_{D_sDK_0^*(800,1430)}$ coupling constant in GeV^{-1} unit.

	$Q^2 = -m_D^2$	$Q^2 = -m_{K_0^*(800)}^2$	Average
$g_{D_sDK_0^*(800)}$	0.97 ± 0.02	0.74 ± 0.05	0.85 ± 0.08

	$Q^2 = -m_D^2$	$Q^2 = -m_{K_0^*(1430)}^2$	Average
$g_{D_sDK_0^*(1430)}$	1.16 ± 0.12	0.49 ± 0.07	0.83 ± 0.09

TABLE III. Value of the $g_{B_sBK_0^*(800,1430)}$ coupling constant in GeV^{-1} unit.

	$Q^2 = -m_B^2$	$Q^2 = -m_{K_0^*(800)}^2$	Average
$g_{B_sBK_0^*(800)}$	2.28 ± 0.18	0.53 ± 0.09	1.14 ± 0.21

	$Q^2 = -m_B^2$	$Q^2 = -m_{K_0^*(1430)}^2$	Average
$g_{B_sBK_0^*(1430)}$	1.85 ± 0.53	0.25 ± 0.04	1.05 ± 0.32

TABLE IV. Value of the $g_{D_s^*DK}$ coupling constant.

	$Q^2 = -m_D^2$	$Q^2 = -m_K^2$	Average
$g_{D_s^*DK}$ (Present work)	2.79 ± 0.24	2.99 ± 0.26	2.89 ± 0.25
$g_{D_s^*DK}$ ([30])	2.72	2.87	2.84 ± 0.31

Table IV also shows the predictions of [30] on $g_{D_s^*DK}$ as the only existing previously calculated coupling constant among the considered vertices. Comparing our results with that of [30], we see a good consistency between two predictions.

Similarly, for B_s^*BK vertex, our result for the pseudoscalar kaon and B off shell is better extrapolated by the exponential fit parametrization

$$g_{B_s^*BK}^{(B)}(Q^2) = 0.66e^{-(Q^2/23.34\,(\mathrm{GeV}^2))} + 0.23, \qquad (59)$$

and in the K off-shell case by the parametrization

$$g_{B_s^*BK}^{(K)}(Q^2) = 4.39e^{-(Q^2/4.02\,(\mathrm{GeV}^2))} - 1.03. \qquad (60)$$

Using the same procedure as above, we find the values depicted in Table V.

In the case of axial-vector kaon, the strong coupling constant obey also the same Boltzmann function as the scalar case. The values of the parameters A_1, A_2, x_0 and Δx for coupling constants in this case are given in Table VI.

TABLE V. Value of the $g_{B_s^*BK}$ coupling constant.

	$Q^2 = -m_B^2$	$Q^2 = -m_K^2$	Average
$g_{B_s^*BK}$	2.40 ± 0.22	3.62 ± 0.34	3.01 ± 0.28

The same procedure as in the scalar and pseudoscalar cases leads to the numerical results for the corresponding coupling constants as presented in Tables VII and VIII.

In summary, the strong coupling constants, $g_{B_sBK_0^*}$, $g_{D_sDK_0^*}$, $g_{B_s^*BK}$, $g_{D_s^*DK}$, $g_{B_s^*BK_1}$, and $g_{D_s^*DK_1}$, have been

TABLE VI. Parameters appearing in the fit function of the coupling constants for the $D_s^*DK_1(1270)$, $D_s^*DK_1(1400)$, $B_s^*BK_1(1270)$, and $B_s^*BK_1(1400)$ vertices. A_1 and A_2 are in GeV^{-1} units, while x_0 and Δx are in the units of GeV^2.

	A_1	A_2	x_0	Δx
$g_{D_s^*DK_1(1270)}^{(D)}(Q^2)$	5.062	−2.337	1.182	1.531
$g_{D_s^*DK_1(1400)}^{(D)}(Q^2)$	73.848	−87.162	118.101	74.590
$g_{D_s^*DK_1(1270)}^{(K_1(1270))}(Q^2)$	0.137	−1.507	6.951	1.845
$g_{D_s^*DK_1(1400)}^{(K_1(1400))}(Q^2)$	−0.106	1.234	6.843	1.847
$g_{B_s^*BK_1(1270)}^{(B)}(Q^2)$	0.764	0.412	11.343	4.708
$g_{B_s^*BK_1(1400)}^{(B)}(Q^2)$	2.463	−2.178	38.732	18.980
$g_{B_s^*BK_1(1270)}^{(K_1(1270))}(Q^2)$	0.047	−23681.595	−31.416	2.914
$g_{B_s^*BK_1(1400)}^{(K_1(1400))}(Q^2)$	−0.021	0.282	3.080	1.233

TABLE VII. Values of the $g_{D_s^* D K_1(1270)}$ and $g_{D_s^* D K_1(1400)}$ coupling constants in GeV^{-1}.

	$Q^2 = -m_D^2$	$Q^2 = -m_{K_1(1270)}^2$	Average
$g_{D_s^* D K_1(1270)}$	2.83 ± 0.09	1.36 ± 0.14	2.09 ± 0.82
	$Q^2 = -m_D^2$	$Q^2 = -m_{K_1(1400)}^2$	Average
$g_{D_s^* D K_1(1400)}$	0.97 ± 0.15	1.12 ± 0.54	1.04 ± 0.78

TABLE VIII. Values of the $g_{B_s^* B K_1(1270)}$ and $g_{B_s^* B K_1(1400)}$ coupling constants in GeV^{-1}.

	$Q^2 = -m_B^2$	$Q^2 = -m_{K_1(1270)}^2$	Average
$g_{B_s^* B K_1(1270)}$	1.18 ± 0.07	0.81 ± 0.45	1.99 ± 0.11
	$Q^2 = -m_B^2$	$Q^2 = -m_{K_1(1400)}^2$	Average
$g_{B_s^* B K_1(1400)}$	0.35 ± 0.05	0.26 ± 0.04	0.30 ± 0.05

calculated in the framework of three-point QCD sum rules. The correlation functions of the considered vertices when both $B(D)$ and $K_0^*(K)(K_1)$ mesons are off shell are evaluated. The final numerical values have been obtained taking the average of the numerical values obtained from both off-shell cases. In the case of the axial vector K_1, which is either $K_1(1270)$ or $K_1(1400)$, the mixing between these two states have also been taken into account. A comparison of the obtained result on $D_s^* DK$ as the only previously calculated coupling constant among the considered strong coupling constants has also been made.

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