

Mass and Meson-Current Coupling Constant of the Tensor $D_2^*(2460)$

H. Sundu¹ *, K. Azizi² †

¹ Department of Physics , Kocaeli University, 41380 Izmit, Turkey

² Department of Physics, Dođuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey

Abstract

We calculate the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson in the framework of QCD sum rules. The obtained result on the mass is compatible with the experimental data.

PACS number(s): 11.55.Hx, 14.40.Lb

*e-mail: hayriye.sundu@kocaeli.edu.tr

†e-mail: kazizi@dogus.edu.tr

1 Introduction

The semileptonic B meson transitions to the orbitally excited charmed mesons as well as the strong transitions of the excited charmed mesons into the other charmed states have been in focus of attention of many Collaborations in the recent years. The BaBar and Belle Collaborations reported the measurement of the products/ratios of the branching fractions of some semileptonic and hadronic decays of the B meson and orbitally excited charmed meson channels [1–3]. Considering these experimental progress and the fact that the decay channels containing orbitally excited charmed meson in the final state supply a considerable contribution to the total semileptonic B meson decay width, more knowledge about the properties of orbitally excited charmed mesons like $D_2^*(2460)$ is needed both experimentally and theoretically.

In the present letter, we calculate the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson with quantum numbers $I(J^P) = 1/2 (2^+)$ in the framework of the QCD sum rules as one of the most applicable and powerful non-perturbative approaches to hadron physics. For details about the method and some of its applications see for instance [4–7]. Our results on the mass and meson-current coupling constant of $D_2^*(2460)$ tensor meson can be used in theoretical calculations on the decays of heavy mesons into orbitally excited charmed $D_2^*(2460)$ meson. Moreover, they may be used to analyze the electromagnetic, semileptonic and strong decay modes of the $D_2^*(2460)$ into other charmed and lighter mesons (For some decay modes of $D_2^*(2460)$ tensor meson and decay modes of heavy mesons into this state see [8]). Some properties such as mass, meson-current coupling constant and electromagnetic multi-poles of the heavy-heavy, light-heavy and light-light tensor mesons have previously calculated using different frameworks. For some of them see [9–14] and references therein. Some semileptonic decays of the B meson into the orbitally excited charmed mesons have also been studied in [15] within the framework of constituent quark model.

The letter is organized as follows. In section II, we briefly present calculations of the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson within the framework of the QCD sum rules. Section III is devoted to the numerical analysis of the considered observables as well as comparison of our result on mass with experimental data.

2 QCD sum rules for mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson

This section is dedicated to calculation of the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson in the framework of the QCD sum rules. The starting point is to consider the following two-point correlation function:

$$\Pi_{\mu\nu,\alpha\beta} = i \int d^4x e^{iq(x-y)} \langle 0 | \mathcal{T} [j_{\mu\nu}(x) \bar{j}_{\alpha\beta}(y)] | 0 \rangle, \quad (1)$$

where, $j_{\mu\nu}$ is the interpolating current of the $D_2^*(2460)$ tensor meson and \mathcal{T} is the time ordering operator. The current $j_{\mu\nu}$ is written in terms of the quark fields as

$$j_{\mu\nu}(x) = \frac{i}{2} \left[\bar{u}(x) \gamma_\mu \overleftrightarrow{D}_\nu(x) c(x) + \bar{u}(x) \gamma_\nu \overleftrightarrow{D}_\mu(x) c(x) \right], \quad (2)$$

where the $\overleftrightarrow{\mathcal{D}}_\mu(x)$ denotes the four-derivative with respect to x acting on the left and right, simultaneously. It is given as

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2} \left[\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x) \right], \quad (3)$$

with,

$$\begin{aligned} \overrightarrow{\mathcal{D}}_\mu(x) &= \overrightarrow{\partial}_\mu(x) - i\frac{g}{2}\lambda^a A_\mu^a(x), \\ \overleftarrow{\mathcal{D}}_\mu(x) &= \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a A_\mu^a(x). \end{aligned} \quad (4)$$

Here, λ^a are the Gell-Mann matrices and $A_\mu^a(x)$ is the external gluon fields. Considering the Fock-Schwinger gauge ($x^\mu A_\mu^a(x) = 0$), these fields are expressed in terms of the gluon field strength tensor

$$A_\mu^a(x) = \int_0^1 d\alpha \alpha x_\beta G_{\beta\mu}^a(\alpha x) = \frac{1}{2} x_\beta G_{\beta\mu}^a(0) + \frac{1}{3} x_\eta x_\beta \mathcal{D}_\eta G_{\beta\mu}^a(0) + \dots \quad (5)$$

The currents contain derivatives with respect to the space-time, hence we consider the two currents at points x and y . After applying the derivatives with respect to y , we will put $y = 0$.

According to the general criteria of the QCD sum rules, the aforementioned correlation function is calculated via two alternative ways: phenomenologically (physical side) and theoretically (QCD side). In physical side, the two-point correlation function is calculated in terms of hadronic degrees of freedom like mass, meson-current coupling constant, etc. The QCD side is obtained in terms of the QCD parameters such as quark masses, quark and gluon condensates, etc. The two-point QCD sum rules is obtained matching coefficients of the same structure representing the tensor mesons from both sides through a dispersion relation and quark-hadron duality assumption. Finally, we apply Borel transformation to stamp down the contributions belong to the higher states and continuum.

2.1 The physical side

In the physical side, the correlation function is obtained inserting complete set of hadronic state having the same quantum numbers as the interpolating current $j_{\mu\nu}$ into Eq. (1). After performing integral over four- x and putting $y = 0$, we obtain the physical side of correlation function as following form:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{\langle 0 | j_{\mu\nu}(0) | D_2^*(2460) \rangle \langle D_2^*(2460) | \bar{j}_{\alpha\beta}(0) | 0 \rangle}{m_{D_2^*(2460)}^2 - q^2} + \dots, \quad (6)$$

where \dots represents contribution of the higher states and continuum. To proceed, we need to know the matrix element $\langle 0 | j_{\mu\nu}(0) | D_2^*(2460) \rangle$, which is defined in terms of the meson-current coupling constant, mass and polarization tensor

$$\langle 0 | j_{\mu\nu}(0) | D_2^*(2460) \rangle = f_{D_2^*(2460)} m_{D_2^*(2460)}^3 \varepsilon_{\mu\nu}. \quad (7)$$

Combining Eq. (6) and Eq. (7) and performing summation over polarization tensor via

$$\varepsilon_{\mu\nu}\varepsilon_{\alpha\beta}^* = \frac{1}{2}T_{\mu\alpha}T_{\nu\beta} + \frac{1}{2}T_{\mu\beta}T_{\nu\alpha} - \frac{1}{3}T_{\mu\nu}T_{\alpha\beta}, \quad (8)$$

with,

$$T_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{D_2^*(2460)}^2}, \quad (9)$$

the final representation of physical side is obtained as

$$\Pi_{\mu\nu,\alpha\beta} = \frac{f_{D_2^*(2460)}^2 m_{D_2^*(2460)}^6}{m_{D_2^*(2460)}^2 - q^2} \left\{ \frac{1}{2}(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \right\} + \text{other structures} + \dots, \quad (10)$$

where, the explicitly written structure gives contribution to the tensor state.

2.2 The QCD side

The correlation function in QCD side, is calculated in deep Euclidean region, $q^2 \ll 0$, by the help of operator product expansion (OPE) where the short and long distance contributions are separated. The short distance effects are calculated using the perturbation theory, while the long distance effects are parameterized in terms of quark and gluon condensates.

Any coefficient of the structure, $\frac{1}{2}(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha})$, in QCD side, i.e. $\Pi(q^2)$, can be written as a dispersion relation

$$\Pi(q^2) = \int ds \frac{\rho(s)}{s - q^2}, \quad (11)$$

where the spectral density is given by the imaginary part of the $\Pi(q^2)$ function, i.e., $\rho(s) = \frac{1}{\pi} \text{Im}[\Pi(s)]$. As we mentioned above, the correlation function contains both perturbative and non-perturbative effects, hence the spectral density can be decomposed as

$$\rho(s) = \rho^{\text{pert}}(s) + \rho^{\text{nonpert}}(s), \quad (12)$$

where, $\rho^{\text{pert}}(s)$ and $\rho^{\text{nonpert}}(s)$ denote the contributions coming from perturbative and non-perturbative effects, respectively.

Now, we proceed to calculate the spectral density $\rho(s)$. Making use of the tensor current presented in Eq. (2) into the correlation function in Eq. (1) and contracting out all quark fields applying the Wick's theorem, we get:

$$\begin{aligned} \Pi_{\mu\nu,\alpha\beta} = & \frac{i}{4} \int d^4x e^{iq(x-y)} \left\{ \text{Tr} \left[S_u(y-x) \gamma_\mu \overleftrightarrow{D}_\nu(x) \overleftrightarrow{D}_\beta(y) S_c(x-y) \gamma_\alpha \right] \right. \\ & \left. + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\}. \end{aligned} \quad (13)$$

To obtain the correlation function from QCD side, we need to know the heavy and light quarks propagators $S_c(x-y)$ and $S_u(x-y)$. These propagators have been calculated in

[6]. Ignoring the gluon fields which have very small contributions in our calculations (see also [9, 10]), their explicit expressions between two points up to quark condensates can be written as

$$S_c^{ij}(x-y) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot (x-y)} \left\{ \frac{\not{k} + m_c}{k^2 - m_c^2} \delta_{ij} + \dots \right\}, \quad (14)$$

and

$$\begin{aligned} S_u^{ij}(x-y) &= i \frac{\not{x} - \not{y}}{2\pi^2(x-y)^4} \delta_{ij} - \frac{m_u}{4\pi^2(x-y)^2} \delta_{ij} - \frac{\langle \bar{u}u \rangle}{12} \left[1 - i \frac{m_u}{4} (\not{x} - \not{y}) \right] \delta_{ij} \\ &- \frac{(x-y)^2}{192} m_0^2 \langle \bar{u}u \rangle \left[1 - i \frac{m_u}{6} (\not{x} - \not{y}) \right] \delta_{ij} + \dots \end{aligned} \quad (15)$$

Note that the gluon condensates are also ignored in [16–18] since their contributions are suppressed by large denominators, so they play minor roles in calculations.

The next step is to put the expressions of the propagators and apply the derivatives with respect to x and y in Eq. (13) and finally set $y = 0$. As a result, the following expression for the QCD side of the correlation function in coordinate space is obtained:

$$\begin{aligned} \Pi_{\mu\nu,\alpha\beta} &= \frac{N_c}{16} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_c^2} \int d^4x e^{i(q-k) \cdot x} \left\{ \left[Tr \Gamma_{\mu\nu,\alpha\beta} \right] + \left[\beta \leftrightarrow \alpha \right] + \left[\nu \leftrightarrow \mu \right] \right. \\ &\left. + \left[\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu \right] \right\}, \end{aligned} \quad (16)$$

where $N_c = 3$ is the color factor and ,

$$\begin{aligned} \Gamma_{\mu\nu,\alpha\beta} &= k_\nu k_\beta \left[\frac{i \not{x}}{2\pi^2 x^4} + \left(\frac{1}{12} + \frac{x^2}{192} m_0^2 \right) \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ &- ik_\nu \left[\frac{i}{2\pi^2} \left(\frac{\gamma_\beta}{x^4} - \frac{4x_\beta \not{x}}{x^6} \right) + \frac{x_\beta}{96} m_0^2 \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ &+ ik_\beta \left[\frac{i}{2\pi^2} \left(\frac{4x_\nu \not{x}}{x^6} - \frac{\gamma_\nu}{x^4} \right) + \frac{x_\nu}{96} m_0^2 \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ &+ \left[\frac{8i}{2\pi^2} \left(\frac{x_\beta x_\nu \not{x}}{x^8} + \frac{1}{2x^6} \left(\delta_\beta^\nu \not{x} - \gamma_\nu x_\beta + \gamma_\beta x_\nu \right) + \frac{\gamma_\nu x_\beta}{x^6} - \frac{4x_\beta x_\nu \not{x}}{x^8} \right) \right. \\ &\left. + \frac{\delta_\beta^\nu}{96} m_0^2 \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu]. \end{aligned} \quad (17)$$

In order to calculate the integrals, first we transform the terms containing $\frac{1}{(x^2)^n}$ to the momentum space ($x \rightarrow p$) and replace $x_\mu \rightarrow -i \frac{\partial}{\partial q_\mu}$. The integral over four- x gives us a Dirac Delta function which help us perform the integral over four- k . The last integral which is over four- p is performed using the Feynman parametrization and the relation

$$\int d^4p \frac{(p^2)^\beta}{(p^2 + L)^\alpha} = \frac{i\pi^2 (-1)^{\beta-\alpha} \Gamma(\beta+2) \Gamma(\alpha-\beta-2)}{\Gamma(2) \Gamma(\alpha) [-L]^{\alpha-\beta-2}}. \quad (18)$$

After dimensional regularization and taking the imaginary part and selecting the coefficient of the aforesaid structure, the spectral densities are obtained as:

$$\rho^{pert}(s) = \frac{N_c}{960 \pi^2 s^3} \left(m_c^2 - s \right)^4 \left(2m_c^2 + 3s \right), \quad (19)$$

and

$$\rho^{nonpert}(s) = -\frac{N_c}{48s} m_c m_0^2 \langle \bar{u}u \rangle. \quad (20)$$

After achieving the correlation function in two different ways, we match these two different representations to obtain two-point QCD sum rules for the meson-current coupling constant and mass. In order to suppress contributions of the higher states and continuum, we apply Borel transformation with respect to the initial momentum squared, q^2 , to both sides of the sum rules and use the quark-hadron duality assumption. As a result, the following sum rule for the meson-current coupling constant of the $D_2^*(2460)$ tensor meson is obtained:

$$f_{D_2^*(2460)}^2 e^{-m_{D_2^*(2460)}^2/M^2} = \frac{1}{m_{D_2^*(2460)}^6} \int_{m_c^2}^{s_0} ds \left(\rho^{pert}(s) + \rho^{nonpert}(s) \right) e^{-s/M^2}, \quad (21)$$

where s_0 is the continuum threshold and M^2 is the Borel mass parameter. Differentiating Eq. (21) with respect to $-\frac{1}{M^2}$ and dividing both sides of the obtained result to both sides of Eq. (21), we obtain the following sum rule for the mass of the $D_2^*(2460)$ tensor meson:

$$m_{D_2^*(2460)}^2 = \frac{\int_{m_c^2}^{s_0} ds \left(\rho^{pert}(s) + \rho^{nonpert}(s) \right) s e^{-s/M^2}}{\int_{m_c^2}^{s_0} ds \left(\rho^{pert}(s) + \rho^{nonpert}(s) \right) e^{-s/M^2}}. \quad (22)$$

3 Numerical results

In this section, we carry out numerical analysis of the sum rules for the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson. The input parameters are taken to be $m_c = (1.27_{-0.09}^{+0.07}) \text{ GeV}$ [8], $\langle \bar{u}u(1 \text{ GeV}) \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$ [19] and $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$ [20]. The sum rules contain also two auxiliary parameters, namely the Borel parameter M^2 and continuum threshold s_0 . According to the standard procedure in QCD sum rules, the physical quantities should be independent of these parameters, hence we shall look for working regions for these parameters such that the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson weakly depend on these helping parameters. The continuum threshold s_0 is not totally arbitrary and it is in correlation with the energy of the first excited state. Since we have no experimental information on the first excited state of the meson under consideration, comparing to the other charmed mesons with similar mass we choose the interval $s_0 = (8.1 \pm 0.5) \text{ GeV}^2$ for the continuum threshold (for more information see [21, 22]). This interval is obtained from $(m_{meson} + 0.3)^2 \leq s_0 \leq (m_{meson} + 0.5)^2$ and our numerical calculations show that in this region, the results of physical observables very weakly depend on the continuum threshold.

The working region for the Borel mass parameter is determined requiring that not only the higher state and continuum contributions are suppressed but also the contributions of the higher order operators are ignorable, i.e. the sum rules are convergent. As a result, the working region for the Borel parameter is found to be $3 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2$. To show how the OPE is convergent and the contributions of the higher states and continuum are

suppressed in the used Borel window, as an example we plot the result of sum rule for the meson-current coupling constant in terms of Borel mass parameter at $s_0 = 8.2 \text{ GeV}^2$ in figure 1. From this figure we see that the main contribution comes from the perturbative part such that the non-perturbative part constitutes only 15% of the total contribution. From this figure it is also clear that not only the contribution of the higher states and continuum is zero but also the higher dimensional operators ($d > 5$) have very small contributions and can be safely ignored.

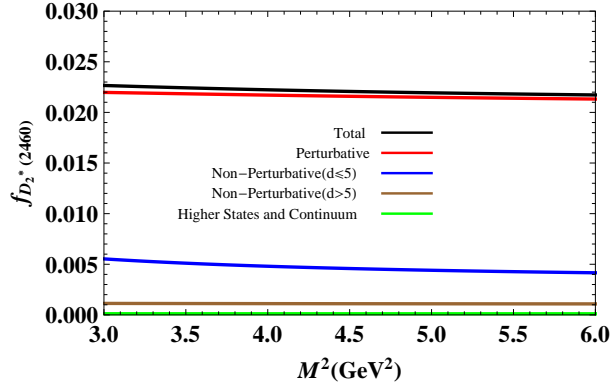


Figure 1: Comparison of the different contributions to the sum rule for the meson-current coupling constant at $s_0 = 8.2 \text{ GeV}^2$.

The dependencies of the mass and total meson-current coupling constant of the $D_2^*(2460)$ tensor meson on Borel mass parameter are also presented in figure 2. From this figure, we see good stability of the observables under consideration with respect to the variation of Borel mass parameter in its working region. Our numerical results on the mass and meson-

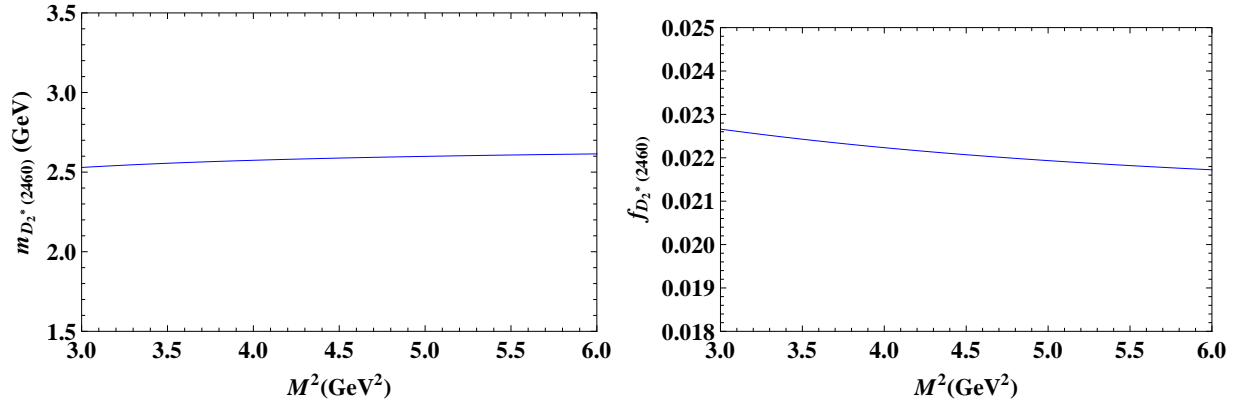


Figure 2: Dependencies of the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson on Borel mass parameter in its working region.

current coupling constant for $D_2^*(2460)$ tensor meson as well as the experimental data on the mass [8] are given in Table 1. The errors quoted in our predictions are due to the variations

| | Present Work | Experiment [8] |
|-------------------|-------------------------------|-----------------------------------|
| $m_{D_2^*(2460)}$ | $(2.53 \pm 0.45) \text{ GeV}$ | $(2.4626 \pm 0.0007) \text{ GeV}$ |
| $f_{D_2^*(2460)}$ | 0.0228 ± 0.0068 | - |

Table 1: Values for the mass and meson-current coupling constant of the $D_2^*(2460)$ tensor meson.

of both auxiliary parameters and uncertainties in input parameters. From Table 1, we see a good consistency between our prediction and the experimental data on the mass of the $D_2^*(2460)$ tensor meson. Any measurement on the meson-current coupling constant and comparison of the result with our prediction can give more information about the nature of the orbitally excited $D_2^*(2460)$ tensor meson.

4 Acknowledgment

We thank Y. Saraç for useful discussions. This work has been supported in part by the scientific and technological research council of Turkey (TUBITAK) under research project No. 110T284.

References

- [1] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 101, 261802 (2008).
- [2] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 103, 051803 (2009).
- [3] D. Liventsev et al. (Belle Collaboration), Phys. Rev. D 77, 091503 (2008).
- [4] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147, 385 (1979).
- [5] M. A. Shifman, A. I. Vainstein, V. I. Zakharov, Nucl. Phys. B 147, 448 (1979).
- [6] L. J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rept. 127, 1 (1985).
- [7] P. Colangelo, A. Khodjamirian, in "At the Frontier of Particle Physics/Handbook of QCD", edited by M. Shifman (World Scientific, Singapore, 2001), Vol. 3, p. 1495.
- [8] K. Nakamura et al., J. Phys. G 37 , 075021, (2010).
- [9] T. M. Aliev, M. A. Shifman, Phys. Lett. B 112 , 401 (1982).
- [10] T. M. Aliev, K. Azizi, V. Bashiry, J. Phys. G, 37, 025001 (2010).
- [11] T. M. Aliev, K. Azizi, M. Savci, Phys. Lett. B 690 (2010) 164.
- [12] Wei Chen, Zi-Xing Cai, Shi-Lin Zhu, arXiv:1107.4949 [hep-ph].

Author's Copy

- [13] T. M. Aliev, K. Azizi, M. Savci, J. Phys. G 37 (2010) 075008.
- [14] F. X. Lee, S. Moerschbacher, W. Wilcox, Phys. Rev. D 78, 094502 (2008).
- [15] J. Segovia, C. Albertus, D. R. Entem, F. Fernandez, E. Hernandez, M.A. Perez-Garcia, Phys. Rev. D 84, 094029 (2011).
- [16] Z. G. Wang, Z. C. Liu, X. H. Zhang, Eur. Phys. J. C 64, 373-386,(2009).
- [17] Z. G. Wang, Nucl. Phys. A 791, 106 (2007).
- [18] Z. G. Wang, W. M. Yang and S. L. Wan, J. Phys. G 31, 971, (2005).
- [19] B. L. Ioffe, Prog. Part. Nucl. Phys. 56, 232 (2006).
- [20] H.G. Dosch, M. Jamin and S. Narison, Phys. Lett. B 220, 251 (1989); V. M. Belyaev, B. L. Ioffe, Sov. Phys. JETP, 57, 716 (1982).
- [21] T. M. Aliev, K. Azizi, A. Ozpineci, Eur. Phys. J. C 51, 593 (2007).
- [22] P. Colangelo, F. De Fazio, and A. Ozpineci, Phys. Rev. D72, 074004 (2005).