

Doubly heavy spin-1/2 baryon spectrum in QCD

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Abstract

We calculate the mass and residue of the heavy spin-1/2 baryons containing two heavy b or c quarks in the framework of QCD sum rules. We use the most general form of the interpolating current in its symmetric and anti-symmetric forms with respect to the exchange of heavy quarks, to calculate the two-point correlation functions describing the baryons under consideration. A comparison of the obtained results with existing predictions from various approaches is also made.

PACS number(s): 11.55.Hx, 14.20.-c, 14.20.Mr, 14.20.Lq

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1 Introduction

The quark model practically describes all phenomena in hadron physics very successfully. This model predicts the existence of hadrons containing two heavy quarks and one can estimate their masses in this framework [1]. The first experimental observation of the doubly heavy baryons was reported by SELEX Collaboration [2], where the double charmed Ξ_{cc}^+ and Ξ_{cc}^{++} baryons were found. This observation was confirmed by measurement of a different weak decay mode [3, 4]. The experimental discovery of the doubly heavy baryons and study of their properties constitutes one of the main directions of the physics program at LHC. (for a review on doubly heavy baryons, see for example [5]).

In this prospect, it would be interesting to present reliable theoretical predictions on properties of these baryons. The masses of the doubly heavy baryons have been estimated in the frameworks of the quark [6–8] and MIT bag [9] models. In order to study the properties of these hadrons in a model independent way, the QCD sum rules method [10] based on QCD Lagrangian is one of the most reliable approaches. In the present work, we calculate the masses and residues of the doubly heavy spin-1/2 baryons within QCD sum rules formalism. We consider both symmetric and anti-symmetric currents with respect to the exchange of heavy quarks defining the baryons under consideration in their most general forms. It should be noted that the masses of doubly heavy baryons with spin-1/2 have been calculated within the same framework using the Ioffe current in [11, 12] and general current in [13]. However, the obtained expressions for mass sum rules in [13] at Ioffe current case does not exactly reduce to those presented in [11, 12] for Ioffe current. The masses of the doubly heavy baryons with spin-3/2 have also been investigated in [13] and [14].

The plan of this work is as follows. In next section, the QCD sum rules for the masses and residues of the doubly spin-1/2 baryons are obtained. Section 3 is devoted to the numerical analysis of the sum rules for physical quantities under consideration. This section encompasses also our comparison of the obtained results with those existing in the literature as well as our concluding remarks.

2 Sum rules for the masses and residues of the doubly heavy baryons with spin-1/2

Before presenting the detailed calculation of sum rules for the masses and residues of the doubly heavy baryons let us discuss the ground states of these baryons in quark model.

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For the ground state of doubly heavy baryons with the identical heavy quarks, $\Xi_{QQ}^{(*)}$ and $\Omega_{QQ}^{(*)}$, the pair of heavy quarks form a diquark with total spin of 1. Adding then the spin-1/2 of light quark, we have two states with total spin-1/2 or spin-3/2. The baryons with (without) star stand for spin 3/2 (1/2) doubly heavy baryons during the text. For these states, the interpolating current should be symmetric with respect to the exchange of the heavy quarks fields. For the states containing two different heavy quarks, in addition to the previous case, i.e., total spin of diquark equal to one, the diquark can also have the total spin zero, which leads to the total spin 1/2 of these states. Obviously, the interpolating current of these states (usually these states are denoted by Ξ'_{bc} and Ω'_{bc}) are anti-symmetric with respect to two heavy quarks fields. In the present work, we deal only with spin-1/2 doubly heavy baryons.

After these preliminary remarks, let us come back to our main problem, i.e., calculation of the masses and residues of the doubly heavy baryons. The interpolating currents here play the role of the wave functions in the quark model. The general expressions of the interpolating currents for the spin-1/2 doubly heavy baryons in their symmetric and anti-symmetric forms can be written as:

$$\begin{aligned}
 \eta^S &= \frac{1}{\sqrt{2}}\epsilon_{abc} \left\{ (Q^{aT} C q^b)\gamma_5 Q^c + (Q'^{aT} C q^b)\gamma_5 Q^c + \beta(Q^{aT} C \gamma_5 q^b)Q^c + \beta(Q'^{aT} C \gamma_5 q^b)Q^c \right\}, \\
 \eta^A &= \frac{1}{\sqrt{6}}\epsilon_{abc} \left\{ 2(Q^{aT} C Q^b)\gamma_5 q^c + (Q^{aT} C q^b)\gamma_5 Q^c - (Q'^{aT} C q^b)\gamma_5 Q^c + 2\beta(Q^{aT} C \gamma_5 Q^b)q^c \right. \\
 &\quad \left. + \beta(Q^{aT} C \gamma_5 q^b)Q^c - \beta(Q'^{aT} C \gamma_5 q^b)Q^c \right\}, \tag{2.1}
 \end{aligned}$$

where β is an arbitrary auxiliary parameter. The case, $\beta = -1$ corresponds to the Ioffe current. Here C stands for the charge conjugation operator, T denotes the transposition, a , b , and c are the color indices; and Q and q correspond to the heavy and light quarks fields, respectively. The interpolating current with the light quark u or d corresponds to the Ξ_{QQq} , and with s to the Ω_{QQq} baryons, respectively. Here we would like to note once more that in the symmetric part, both heavy quarks may be identical or different, but in the anti-symmetric part two heavy quarks must be different.

For calculation of the masses and residues of the doubly heavy baryons within the QCD sum rules, we consider the following correlation function:

$$\Pi^{S(A)}(q) = i \int d^4x e^{iqx} \langle 0 | \mathcal{T} \left\{ \eta^{S(A)}(x) \bar{\eta}^{S(A)}(0) \right\} | 0 \rangle, \tag{2.2}$$

where q is four-momentum of the doubly heavy baryon. This correlation function can be

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written in terms of two independent structures

$$\Pi^{S(A)}(q) = \not{q}\Pi_1^{S(A)}(q^2) + U\Pi_2^{S(A)}(q^2), \quad (2.3)$$

where U stands for unit matrix.

The above mentioned correlation function can be calculated in two different manners. From one side, it is calculated in terms of hadronic parameters called physical or phenomenological part. On the other side, it is evaluated in terms of quarks and gluons and their interactions with each other and QCD vacuum called theoretical or QCD side. Equating these two different representations of the correlation function to each other under the flag of the quark-hadron duality assumption, we get sum rules for the physical quantities under consideration. To suppress the contributions coming from the higher states and continuum and obtain contribution of the ground state, we apply the Borel transformation and continuum subtraction after performing the Fourier integrals. These procedures bring two auxiliary parameters so called Borel mass parameter and continuum threshold, which we shall also find their working region to numerically analyze the obtained sum rules in the next section.

Saturating the correlation function by a complete set of hadronic states with the same quantum numbers as the interpolating current and isolating the ground state baryons in phenomenological side, we have

$$\Pi^{S(A)}(q) = \frac{\langle 0|\eta^{S(A)}(0)|B(q)\rangle\langle B(q)|\bar{\eta}^{S(A)}(0)|0\rangle}{q^2 - m_B^2} + \dots, \quad (2.4)$$

where dots refers to the contribution of the higher states and continuum. The matrix element $\langle 0|\eta(0)|B(q)\rangle$ for spin-1/2 baryons is determined as

$$\langle 0|\eta(0)|B(q, s)\rangle = \lambda_B u(q, s), \quad (2.5)$$

where λ_B is the residue and $u(q, s)$ is the Dirac spinor with s stands for the spin. Putting the above equations all together, and performing summation over spins, we get the following final representation for the phenomenological side:

$$\Pi^{S(A)}(q) = \frac{\lambda_{B^{S(A)}}^2(\not{q} + m_{B^{S(A)}})}{q^2 - m_{B^{S(A)}}^2} + \dots, \quad (2.6)$$

where we have only two independent Lorentz structures \not{q} and U discussed above.

In QCD side, the correlation function is calculated in deep Euclidean region with the help of operator product expansion (OPE). Using the Wick theorem and performing all

contractions of the quarks fields for the symmetric part, we get the following expression in terms of heavy and light quarks propagators:

$$\begin{aligned}
\Pi^S(q) = & iA\epsilon_{abc}\epsilon_{a'b'c'} \int d^4x e^{iqx} \langle 0 | \left\{ -\gamma_5 S_Q^{cb'} S_q^{tba'} S_{Q'}^{ac'} \gamma_5 - \gamma_5 S_{Q'}^{cb'} S_q^{tba'} S_Q^{ac'} \gamma_5 \right. \\
& + \gamma_5 S_{Q'}^{cc'} \gamma_5 Tr \left[S_Q^{ab'} S_q^{tba'} \right] + \gamma_5 S_Q^{cc'} \gamma_5 Tr \left[S_{Q'}^{ab'} S_q^{tba'} \right] \\
& + \beta \left(-\gamma_5 S_Q^{cb'} \gamma_5 S_q^{tba'} S_{Q'}^{ac'} - \gamma_5 S_{Q'}^{cb'} \gamma_5 S_q^{tba'} S_Q^{ac'} - S_Q^{cb'} S_q^{tba'} \gamma_5 S_{Q'}^{ac'} \gamma_5 \right. \\
& - S_{Q'}^{cb'} S_q^{tba'} \gamma_5 S_Q^{ac'} \gamma_5 + \gamma_5 S_{Q'}^{cc'} Tr \left[S_Q^{ab'} \gamma_5 S_q^{tba'} \right] + S_{Q'}^{cc'} \gamma_5 Tr \left[S_Q^{ab'} S_q^{tba'} \gamma_5 \right] \\
& + \gamma_5 S_Q^{cc'} Tr \left[S_{Q'}^{ab'} \gamma_5 S_q^{tba'} \right] + S_Q^{cc'} \gamma_5 Tr \left[S_{Q'}^{ab'} S_q^{tba'} \gamma_5 \right] \left. \right) \\
& + \beta^2 \left(-S_Q^{cb'} \gamma_5 S_q^{tba'} \gamma_5 S_{Q'}^{ac'} - S_{Q'}^{cb'} \gamma_5 S_q^{tba'} \gamma_5 S_Q^{ac'} \right. \\
& \left. + S_{Q'}^{cc'} Tr \left[S_q^{ba'} \gamma_5 S_Q^{tab'} \gamma_5 \right] + S_Q^{cc'} Tr \left[S_q^{ba'} \gamma_5 S_{Q'}^{tab'} \gamma_5 \right] \right) \left. \right\} | 0 \rangle, \tag{2.7}
\end{aligned}$$

where $S' = CS^T C$. In the case of $Q \neq Q'$, the constant A in the above equation takes the value $A = \frac{1}{2}$, while when $Q = Q'$ we have $A = 1$ as a result of extra contractions between the same quark fields. For the anti-symmetric part we have

$$\begin{aligned}
\Pi^A(q) = & \frac{i}{6} \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x e^{iqx} \langle 0 | \left\{ 2\gamma_5 S_Q^{cb'} S_{Q'}^{taa'} S_q^{bc'} \gamma_5 + \gamma_5 S_Q^{cb'} S_q^{tba'} S_{Q'}^{ac'} \gamma_5 \right. \\
& - 2\gamma_5 S_{Q'}^{ca'} S_Q^{tab'} S_q^{bc'} \gamma_5 + \gamma_5 S_{Q'}^{cb'} S_q^{tba'} S_Q^{ac'} \gamma_5 \\
& - 2\gamma_5 S_q^{ca'} S_Q^{tab'} S_{Q'}^{bc'} \gamma_5 + 2\gamma_5 S_q^{ca'} S_{Q'}^{tbb'} S_Q^{ac'} \gamma_5 + 4\gamma_5 S_q^{cc'} \gamma_5 Tr \left[S_Q^{ab'} S_{Q'}^{tba'} \right] \\
& + \gamma_5 S_{Q'}^{cc'} \gamma_5 Tr \left[S_Q^{ab'} S_q^{tba'} \right] + \gamma_5 S_Q^{cc'} \gamma_5 Tr \left[S_{Q'}^{ab'} S_q^{tba'} \right] \\
& + \beta \left(2\gamma_5 S_Q^{cb'} \gamma_5 S_{Q'}^{taa'} S_q^{bc'} + \gamma_5 S_Q^{cb'} \gamma_5 S_q^{tba'} S_{Q'}^{ac'} - 2\gamma_5 S_{Q'}^{ca'} \gamma_5 S_Q^{tab'} S_q^{bc'} \right. \\
& + \gamma_5 S_{Q'}^{cb'} \gamma_5 S_q^{tba'} S_Q^{ac'} - 2\gamma_5 S_q^{ca'} \gamma_5 S_Q^{tab'} S_{Q'}^{bc'} + 2\gamma_5 S_q^{ca'} \gamma_5 S_{Q'}^{tbb'} S_Q^{ac'} \\
& + 2S_Q^{cb'} S_{Q'}^{taa'} \gamma_5 S_q^{bc'} \gamma_5 + S_Q^{cb'} S_q^{tba'} \gamma_5 S_{Q'}^{ac'} \gamma_5 - 2S_{Q'}^{ca'} S_Q^{tab'} \gamma_5 S_q^{bc'} \gamma_5 \\
& + S_{Q'}^{cb'} S_q^{tba'} \gamma_5 S_Q^{ac'} \gamma_5 - 2S_q^{ca'} S_Q^{tab'} \gamma_5 S_{Q'}^{bc'} \gamma_5 + 2S_q^{ca'} S_{Q'}^{tbb'} \gamma_5 S_Q^{ac'} \gamma_5 \\
& + 4\gamma_5 S_q^{cc'} Tr \left[S_Q^{ab'} \gamma_5 S_{Q'}^{tba'} \right] + 4S_q^{cc'} \gamma_5 Tr \left[S_Q^{ab'} S_{Q'}^{tba'} \gamma_5 \right] + \gamma_5 S_{Q'}^{cc'} Tr \left[S_Q^{ab'} \gamma_5 S_q^{tba'} \right] \\
& + S_{Q'}^{cc'} \gamma_5 Tr \left[S_Q^{ab'} S_q^{tba'} \gamma_5 \right] + \gamma_5 S_Q^{cc'} Tr \left[S_{Q'}^{ab'} \gamma_5 S_q^{tba'} \right] + S_Q^{cc'} \gamma_5 Tr \left[S_{Q'}^{ab'} S_q^{tba'} \gamma_5 \right] \left. \right) \\
& + \beta^2 \left(2S_Q^{cb'} \gamma_5 S_{Q'}^{taa'} \gamma_5 S_q^{bc'} + S_Q^{cb'} \gamma_5 S_q^{tba'} \gamma_5 S_{Q'}^{ac'} - 2S_{Q'}^{ca'} \gamma_5 S_Q^{tab'} \gamma_5 S_q^{bc'} \right. \\
& + S_{Q'}^{cb'} \gamma_5 S_q^{tba'} \gamma_5 S_Q^{ac'} - 2S_q^{ca'} \gamma_5 S_Q^{tab'} \gamma_5 S_{Q'}^{bc'} + 2S_q^{ca'} \gamma_5 S_{Q'}^{tbb'} \gamma_5 S_Q^{ac'} \\
& \left. + 4S_q^{cc'} Tr \left[S_{Q'}^{tba'} \gamma_5 S_Q^{tab'} \gamma_5 \right] + S_{Q'}^{cc'} Tr \left[S_q^{ba'} \gamma_5 S_Q^{tab'} \gamma_5 \right] + S_Q^{cc'} Tr \left[S_q^{ba'} \gamma_5 S_{Q'}^{tab'} \gamma_5 \right] \right) \left. \right\} | 0 \rangle. \tag{2.8}
\end{aligned}$$

For calculation of the QCD side, we need to know the explicit expressions of the light

and heavy quarks propagators. Their expressions in coordinate space are presented in the Appendix.

The coefficient of any structure in Eq. (2.3) in QCD side can be written as the following dispersion integral:

$$\Pi_{1(2)}^{S(A)}(q^2) = \int \frac{\rho_{1(2)}^{S(A)}(s)}{s - q^2} ds, \quad (2.9)$$

where $\rho_{1(2)}^{S(A)}$ are spectral densities which can be obtained from imaginary parts of the correlation functions, i.e.,

$$\rho_{1(2)}^{S(A)}(s) = \frac{1}{\pi} \text{Im} \left\{ \Pi_{1(2)}^{S(A)}(s) \right\}, \quad (2.10)$$

where subindex 1(2) corresponds to the coefficient of the structure $\not{q}(U)$.

Our main task now is to calculate these spectral densities. For this aim, we write integral representation of the Bessel functions in Euclidean space,

$$\frac{K_\nu(m_Q \sqrt{-x^2})}{(\sqrt{-x^2})^\nu} = \frac{1}{2} \int \frac{dt}{t^{\nu+1}} e^{-\frac{m_Q}{2}(t + \frac{x_E^2}{t})}, \quad (2.11)$$

where $x_E^2 = -x^2$. We also use the Schwinger representation for the terms containing $\frac{1}{(x^2)^n}$ to write them as exponential form after Wick rotation, i.e.,

$$\frac{1}{(x_E^2)^n} = \frac{1}{\Gamma(n)} \int_0^\infty d\lambda \lambda^{n-1} e^{-\lambda x_E^2}. \quad (2.12)$$

Then, we perform the Gaussian integrals $d^D x_E$ and make dimensional regularization. After lengthy calculations for the spectral densities, we get the results presented in the Appendix.

As has already been noted, the mass sum rules for the heavy baryons are also calculated in [11, 12] and [13]. Here we should remind that although the general form of the interpolating current is used in [13] similar to our case, the results are presented only for the symmetric current case and the anti-symmetric current with respect to the heavy quarks is not analyzed. On the other hand, the results presented in [11] and [12] are obtained only for the Ioffe current ($\beta = -1$). Here we would like to compare our results that are obtained using the symmetric current with those presented in [13] as well as our predictions in the case of special Ioffe current with those of for instance [11] in details.

Comparing our results on the spectral densities for the symmetric current case with those given in [13], we see that as far as the \not{q} structure is considered, the perturbative parts coincide, but there is a sign difference in the terms containing $\langle \bar{q}q \rangle$. In the case of

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the structure U , the situation is reversed, i.e., the signs of the perturbative parts in two works are different while the terms containing $\langle \bar{q}q \rangle$ have the same sign. Moreover, the terms containing the quark condensates multiplied by m_q factor also show the same differences. For both structures, the coefficients of the operator $m_0^2 \langle \bar{q}q \rangle$ in our case are totally different from those presented in [13].

Comparing our results for the Ioffe case and symmetric current with the results of [11], we observe that the coefficient of the $m_Q m_{Q'}$ factor in spectral density $\rho_1^S(s)$ in [11] contains an extra multiplying factor of 4. Moreover, the quark condensate term is multiplied by the factor $5/8$, while in our case this factor is $1/2$.

Now let us compare the results for the spectral density $\rho_2^S(s)$. In our case, there is no any term multiplied by only the light quark mass m_q (without condensates), while in [11] there exists such a term (first term in Eq. (26) in [11]). When we check the term containing the quark condensate multiplied by $m_Q m_{Q'}$, our result is half of that given in [11].

In the anti-symmetric current case, we obtain the following differences among our results compared to those given in [11].

For the spectral density $\rho_1^A(s)$:

- the coefficient of the term $m_Q m_{Q'}$ in [11] contains an extra factor of 2.
- The quark condensate term in our case is multiplied by $1/4$, but it is multiplied by $-5/32$ in [11].
- We have the term,

$$\frac{1}{12} \langle \bar{q}q \rangle \left[(1 - \alpha) m_Q + \alpha m_{Q'} \right],$$

in our result, which is totally absent in [11].

- Also, the term

$$\frac{1}{16} \frac{m_q}{\alpha \beta} (\beta m_{Q'} + \alpha m_Q),$$

is absent in [11].

When the spectral density $\rho_2^A(s)$ is analyzed, we see that

- the term,

$$\frac{1}{32\pi^4} \frac{1}{\alpha^2 \beta^2} (\alpha m_{Q'} + \beta m_Q),$$

appears in our case, but is absent in [11].

- The coefficients of the operators $\langle \bar{q}q \rangle$ and $m_0^2 \langle \bar{q}q \rangle$ in [11] are also different compared to our results.

Our final task in this section is to equate the coefficients of the selected structures from both physical and QCD sides to obtain QCD sum rules for the mass and residue of the doubly heavy baryons under consideration. After performing Borel transformation with respect to the variable- q^2 as well as continuum subtraction to suppress contribution of the higher states and continuum; and using quark-hadron duality assumption we get

$$\begin{aligned} \lambda_{B^{S(A)}}^2 e^{\frac{-m_{B^{S(A)}}^2}{M^2}} &= \int_{(m_Q+m_{Q'})^2}^{s_0} ds \rho_1^{S(A)}(s) e^{\frac{-s}{M^2}}, \\ \lambda_{B^{S(A)}}^2 m_{B^{S(A)}} e^{\frac{-m_{B^{S(A)}}^2}{M^2}} &= \int_{(m_Q+m_{Q'})^2}^{s_0} ds \rho_2^{S(A)}(s) e^{\frac{-s}{M^2}}, \end{aligned} \quad (2.13)$$

where, M and s_0 are Borel mass parameter and continuum threshold, respectively. Obviously, the mass sum rule for the baryons under consideration is obtained eliminating the residue in each of the above equations. This is possible applying derivative with respect to the $-\frac{1}{M^2}$ to both sides of each equation in Eq. (2.13) and dividing by themselves. As a result, we get

$$m_{B^{S(A)}}^2 = \frac{\int_{(m_Q+m_{Q'})^2}^{s_0} ds s \rho_i^{S(A)}(s) e^{\frac{-s}{M^2}}}{\int_{(m_Q+m_{Q'})^2}^{s_0} ds \rho_i^{S(A)}(s) e^{\frac{-s}{M^2}}}, \quad (2.14)$$

where, i can be either 1 or 2.

3 Numerical results

In this section, we present our numerical results on the mass and residues of the doubly heavy spin-1/2 baryons. For the heavy quark masses, we use their $\overline{\text{MS}}$ masses, which are given as, $\bar{m}_c(\bar{m}_c) = (1.28 \pm 0.03) \text{ GeV}$, $\bar{m}_b(\bar{m}_b) = (4.16 \pm 0.03) \text{ GeV}$ [15] and $m_s(2 \text{ GeV}) = (102 \pm 8) \text{ GeV}$ [16]. The values of the quark condensates are taken as $\bar{u}u(1 \text{ GeV}) = \bar{d}d(1 \text{ GeV}) = -(246_{-19}^{+28} \text{ MeV})^3$ [17], $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$ and $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$. It should be noted here that in our calculations we neglect the gluon condensate contributions, since their influence to the total result is quite small due to the fact that we have two loops in perturbative part and radiation of gluons from heavy quark propagators contains an extra α_s and $1/m_Q$ suppression. This expectation is confirmed by explicit calculation in [12], and it is shown that the gluon contribution constitutes only about 2% of the total result. Therefore, gluon condensate contribution can safely be neglected. Here we should also

stress that the two different structures under consideration give approximately the same results, hence in the following, we present the results extracted for the structure \not{q} .

The sum rules for physical quantities contain also three auxiliary parameters, namely Borel mass parameter M^2 continuum threshold s_0 and general parameter β entered the general spin-1/2 currents. We shall find their working region such that the masses and residues are practically independent of these parameters according to the standard criteria in QCD sum rules.

The continuum threshold s_0 is not totally arbitrary but depends on the energy of the first excited state. As we have not enough information about the first excited states of these baryons, we take the value of the continuum threshold to be in the intervals $s_0 = (108 - 120) \text{ GeV}^2$ for bb , $s_0 = (45 - 56) \text{ GeV}^2$ for bc and $s_0 = (13 - 21) \text{ GeV}^2$ for cc baryons in accordance with the baryons containing a single heavy quark. Note that there is an approach which introduces the effective thresholds, i.e., it is assumed that the continuum threshold s_0 is dependent on Q^2 [18]. But in the present work we will follow the standard procedure, i.e., s_0 is independent of Q^2 . Our numerical results show that in the presented intervals for different doubly heavy baryons, the physical quantities under consideration weakly depend on the continuum threshold.

The upper bound on the Borel mass parameter M^2 is obtained from the condition that the pole contribution is larger compared to continuum and higher states. For this aim, we consider the ratio,

$$R = \frac{\int_{(m_Q+m_{Q'})^2}^{s_0} \rho(s) e^{-s/M^2}}{\int_{(m_Q+m_{Q'})^2}^{\infty} \rho(s) e^{-s/M^2}}, \quad (3.15)$$

which describes the relative contributions of the continuum and pole. Demanding that $R > 1/2$ (i.e., the pole contribution exceeds the contributions of the higher states and continuum), we obtain the maximum values of M^2 for different channels as follows:

$$M_{max}^2 = \begin{cases} 6 \text{ GeV}^2 & (\text{at } \sqrt{s_0} = 4.2 \text{ GeV}), \text{ for } \Xi_{cc} \text{ and } \Omega_{cc} \\ 9 \text{ GeV}^2 & (\text{at } \sqrt{s_0} = 7.5 \text{ GeV}), \text{ for } \Xi_{bc} \text{ and } \Omega_{bc} \\ 15 \text{ GeV}^2 & (\text{at } \sqrt{s_0} = 10.9 \text{ GeV}), \text{ for } \Xi_{bb} \text{ and } \Omega_{bb}. \end{cases} \quad (3.16)$$

The lower bound on M^2 is determined from the condition that the perturbative contribution should be larger compared to the nonperturbative contribution. As a result of the

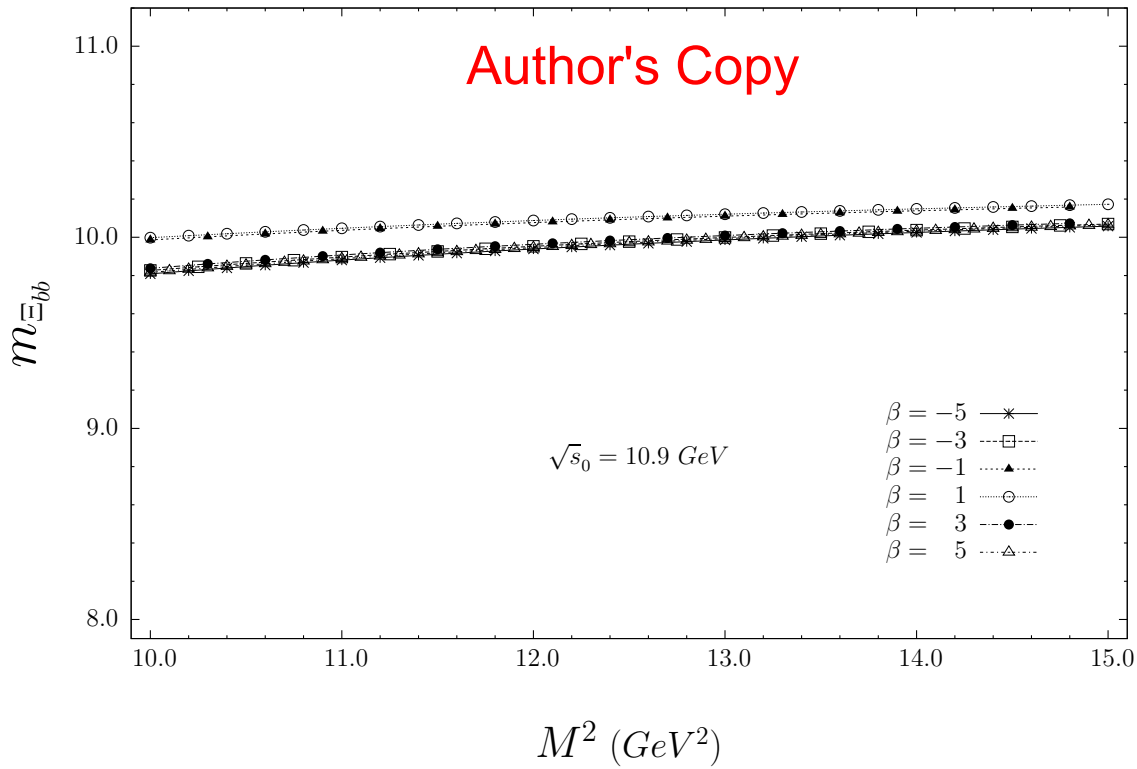


Figure 1: The dependence of the mass of the Ξ_{bb} baryon on M^2 at the fixed value of $\sqrt{s_0} = 10.9 \text{ GeV}$ and at six different values of the parameter β .

analysis of this restriction we get,

$$M_{min}^2 = \begin{cases} 3 \text{ GeV}^2, & \text{for } \Xi_{cc} \text{ and } \Omega_{cc} \\ 6 \text{ GeV}^2, & \text{for } \Xi_{bc} \text{ and } \Omega_{bc} \\ 10 \text{ GeV}^2, & \text{for } \Xi_{bb} \text{ and } \Omega_{bb}, \end{cases} \quad (3.17)$$

at aforementioned values of $\sqrt{s_0}$.

Our numerical analysis shows that in these “working regions” of M^2 , the perturbative contributions also exceed the nonperturbative contributions. For example, for the Ξ_{bb} baryon, at $\sqrt{s_0} = 10.9 \text{ GeV}$ and at $M^2 = 11 \text{ GeV}^2$ the contribution from perturbative part constitutes about 70% of the total result, while the higher states and nonperturbative contributions constitute about 30% of the total result. Similar results are observed for all considered baryons.

As an example in Fig. (1) we present the dependence of the mass of the Ξ_{bb} baryon on M^2 at the fixed value of $\sqrt{s_0} = 10.9 \text{ GeV}$ and at six different values of the parameter β . From this figure, we see that the mass of the Ξ_{bb} baryon exhibits a good stability with respect to the variation in M^2 .

Having determined the working region of M^2 and the values of $\sqrt{s_0}$, our final attempt is to find the working region of the arbitrary parameter β . Depicted in Figure (2) is

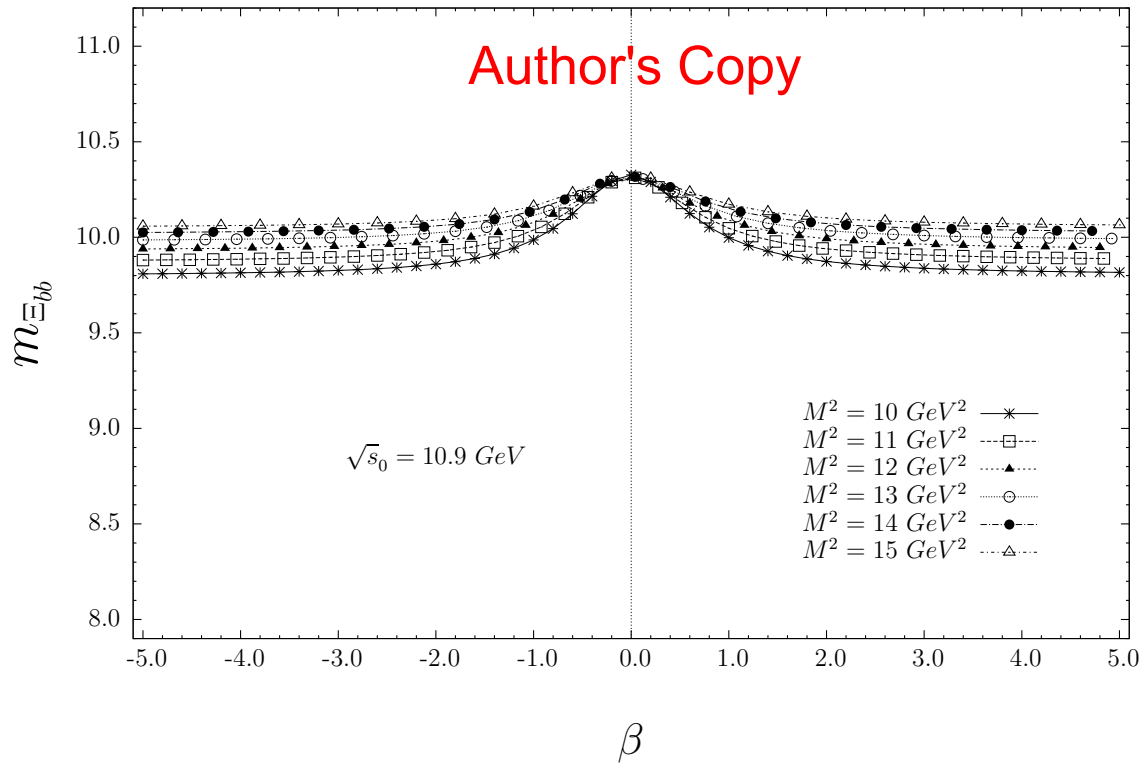


Figure 2: The dependence of the mass of the Ξ_{bb} baryon on β at the fixed value of $\sqrt{s_0} = 10.9 \text{ GeV}$ and at six different values of M^2 .

the dependence of the mass of the Ξ_{bb} baryon on β . We see from this figure that when $\beta \geq 2$ and $\beta \leq -2$ the mass of the Ξ_{bb} baryon depends very weakly on β and we obtain $m_{\Xi_{bb}} = 9.96 \pm 0.90 \text{ GeV}$ at $M^2 = 11 \text{ GeV}^2$ and $\sqrt{s_0} = 10.9 \text{ GeV}$.

Performing similar analysis for other baryons by using the above-mentioned regions for the auxiliary parameters, we obtain the numerical values of their masses which are presented in Table 1.

Baryon	M^2	$\sqrt{s_0}$	This work	[11]	[12]	[13]	[6]	[19]	Exp [20]
Ξ_{bb}	11.0	10.9	9.96(0.90)	9.78(0.07)	10.17(0.14)	9.94(0.91)	10.202	—	—
Ω_{bb}	11.0	10.9	9.97(0.90)	9.85(0.07)	10.32(0.14)	9.99(0.91)	10.359	—	—
Ξ_{bc}	8.0	7.5	6.72(0.20)	6.75(0.05)	—	6.86	6.933	7.053	—
Ω_{bc}	8.0	7.5	6.75(0.30)	7.02(0.08)	—	6.864	7.088	7.148	—
Ξ_{cc}	5.0	4.6	3.72(0.20)	4.26(0.19)	3.57(0.14)	3.52(0.06)	3.620	3.676	3.5189(0.0009)
Ω_{cc}	5.0	4.6	3.73(0.20)	4.25(0.20)	3.71(0.14)	3.53(0.06)	3.778	3.787	—
Ξ'_{bc}	8.0	7.5	6.79(0.20)	6.95(0.08)	—	—	6.963	7.062	—
Ω'_{bc}	8.0	7.5	6.80(0.30)	7.02(0.08)	—	—	7.116	7.151	—

Table 1: The mass of the doubly heavy spin-1/2 baryons (in units of GeV) at $\beta = \pm 2$.

For comparison, we also present predictions of some other theoretical papers as well as existing experimental data on mass of Ξ_{cc} doubly charmed baryon. The errors in the values of the present work also belong to the uncertainties in determination of the working regions for different auxiliary parameters, as well as uncertainties in the values of the input parameters. From Table 1, We observe that different approaches predict very close results on the masses of the doubly heavy baryons. Note that although there are some discrepancies between our results on the expressions of the spectral densities with those of the [11, 12] and [13] discussed in the previous section, considering the uncertainties of the values presented in this Table, there is a good consistency between our numerical values and those existing in [11, 12] and [13] which apply the same approach to calculate the masses of the doubly heavy baryons.

We also present the numerical values for the residues of the doubly heavy baryons in the present work together with the existing prediction of [11] and [12] in Table 2. The values presented inside the parenthesis show the uncertainties of the results on the residues. In obtaining the values of residues of the doubly heavy baryons, we use the values of M^2 and $\sqrt{s_0}$ that are presented in Table 1, at $\beta = \pm 2$. It follows from Table 2 that our predictions on the residues are quite close to the results of [12], except the values of residues for Ξ_{bb} and Ω_{bb} . But our predictions and the predictions of [12] on the residues, are considerably different compared to the ones given in [11].

Baryon	Present work	[11]	[12]
Ξ_{bb}	0.44(0.08)	$0.067 \div 0.057$	0.252(0.064)
Ω_{bb}	0.45(0.08)	—	0.311(0.077)
Ξ_{bc}	0.28(0.05)	$0.046 \div 0.021$	—
Ω_{bc}	0.29(0.05)	—	—
Ξ_{cc}	0.16(0.03)	$0.042 \div 0.026$	0.115(0.027)
Ω_{cc}	0.18(0.04)	—	0.138(0.030)
Ξ'_{bc}	0.30(0.05)	—	—
Ω'_{bc}	0.31(0.06)	—	—

Table 2: The residues of the doubly heavy spin-1/2 baryons (in units of GeV^3).

At the end of this section we would like to mention that according to Eq. (2.13), there

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is another way for determination of the masses of the heavy baryon, i.e.,

$$m_{B^{S(A)}} = \frac{\int_{(m_Q+m_{Q'})^2}^{s_0} ds \rho_2^{S(A)}(s) e^{\frac{-s}{M^2}}}{\int_{(m_Q+m_{Q'})^2}^{s_0} ds \rho_1^{S(A)}(s) e^{\frac{-s}{M^2}}}. \quad (3.18)$$

Our numerical calculations show that, using this formula, the central values of the masses presented in Table 1 are changed maximally 5%.

In summary, we have calculated the masses and residues of the doubly heavy spin-1/2 baryons considering both symmetric and anti-symmetric currents with respect to the exchange of heavy quarks in their most general forms in the framework of QCD sum rules. Our results are in a good consistency with the predictions of the other theoretical papers as well as the experimental data on the mass of the doubly-charmed Ξ_{cc} baryon. We hope that the LHC will provide possibility to study the doubly heavy baryons in near future.

4 Appendix

In this appendix, we present the expressions of the light and heavy quarks propagators as well as the spectral densities.

In our calculations, we have used the following expression for the light quark propagator:

$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle\bar{q}q\rangle}{12} \left(1 - i\frac{m_q}{4}\not{x}\right) - \frac{x^2}{192} m_0^2 \langle\bar{q}q\rangle \left(1 - i\frac{m_q}{6}\not{x}\right). \quad (4.19)$$

The heavy quark propagator in an external field is given as

$$\begin{aligned} S_Q(x) &= \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_Q^2\not{x}}{4\pi^2 x^2} K_2(m_Q\sqrt{-x^2}) \\ &- ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 du \left[\frac{\not{k} + m_Q}{2(m_Q^2 - k^2)^2} G^{\mu\nu}(ux) \sigma_{\mu\nu} + \frac{u}{m_Q^2 - k^2} x_\mu G^{\mu\nu} \gamma_\nu \right], \end{aligned} \quad (4.20)$$

where K_1 and K_2 are the modified Bessel function of the second kind.

The explicit expressions for the spectral densities obtained after procedure mentioned in the body text are given as:

$$\begin{aligned} \rho_1^S(s) &= \frac{A}{128\pi^4} \int_{\psi_{min}}^{\psi_{max}} \int_{\eta_{min}}^{\eta_{max}} d\psi d\eta \left\{ 3\mu \left[\psi\eta \left[5 + \beta(2 + 5\beta) \right] \mu + 2(-1 + \psi + \eta)(-1 + \beta)^2 m_Q m_{Q'} \right. \right. \\ &- \left. \left. 6(-1 + \beta^2) m_q (\eta m_Q + \psi m_{Q'}) \right] \right\} + \frac{A\langle\bar{q}q\rangle}{16\pi^2} \int_{\psi_{min}}^{\psi_{max}} d\psi \left\{ - \left[(-1 + \psi)\psi \left[5 + \beta(2 + 5\beta) \right] m_q \right] \right. \\ &\left. + 3(-1 + \beta^2) \left[(-1 + \psi) m_Q - \psi m_{Q'} \right] \right\}, \end{aligned} \quad (4.21)$$

$$\begin{aligned} \rho_1^A(s) &= \frac{1}{256\pi^4} \int_{\psi_{min}}^{\psi_{max}} \int_{\eta_{min}}^{\eta_{max}} d\psi d\eta \left\{ \mu \left[3\psi\eta \left[5 + \beta(2 + 5\beta) \right] \mu + 2(-1 + \beta) \left[(-1 + \psi + \eta) \right. \right. \right. \\ &\left. \left. \left. + 11\beta \right] m_Q m_{Q'} - (1 + 5\beta) m_q (\eta m_Q + \psi m_{Q'}) \right] \right\} \\ &+ \frac{\langle\bar{q}q\rangle}{96\pi^2} \int_{\psi_{min}}^{\psi_{max}} d\psi \left\{ - 3(-1 + \psi)\psi \left[5 + \beta(2 + 5\beta) \right] m_q \right. \\ &\left. + (-1 + \beta)(1 + 5\beta) \left[(-1 + \psi) m_Q - \psi m_{Q'} \right] \right\}, \end{aligned} \quad (4.22)$$

$$\begin{aligned}
 \rho_2^S(s) &= \frac{A}{128\pi^4} \int_{\psi_{min}}^{\psi_{max}} \int_{\eta_{min}}^{\eta_{max}} d\psi d\eta \left\{ 3\mu \left[3\psi(-1 + \beta^2)\mu m_{Q'} + m_Q \left[3\eta(-1 + \beta^2)\mu \right. \right. \right. \\
 &- \left. \left. \left. 2 \left[5 + \beta(2 + 5\beta) \right] m_q m_{Q'} \right] \right] \right\} + \frac{A\langle \bar{q}q \rangle}{32\pi^2} \int_{\psi_{min}}^{\psi_{max}} d\psi \left\{ - \left[(-1 + \psi)\psi(-1 + \beta)^2 \left[3m_0^2 \right. \right. \right. \\
 &+ \left. \left. \left. 4\mu' - 2s \right] + 2 \left[5 + \beta(2 + 5\beta) \right] m_Q m_{Q'} + 6(-1 + \beta^2)m_q \left[(-1 + \psi)m_Q - \psi m_{Q'} \right] \right] \right. \\
 &\left. - \frac{3}{4} m_0^2 (1 - \beta)^2 \right\}, \tag{4.23}
 \end{aligned}$$

$$\begin{aligned}
 \rho_2^A(s) &= \frac{1}{256\pi^4} \int_{\psi_{min}}^{\psi_{max}} \int_{\eta_{min}}^{\eta_{max}} d\psi d\eta \left\{ \mu \left[\psi(-1 + \beta)(1 + 5\beta)\mu m_{Q'} + m_Q \left[\eta(-1 + \beta)(1 + 5\beta)\mu \right. \right. \right. \\
 &- \left. \left. \left. 6 \left[5 + \beta(2 + 5\beta) \right] m_q m_{Q'} \right] \right] \right\} + \frac{\langle \bar{q}q \rangle}{192\pi^2} \int_{\psi_{min}}^{\psi_{max}} d\psi \left\{ - \left[(-1 + \psi)\psi(-1 + \beta)(13 + 11\beta) \left[3m_0^2 \right. \right. \right. \\
 &+ \left. \left. \left. 4\mu' - 2s \right] + 6 \left[5 + \beta(2 + 5\beta) \right] m_Q m_{Q'} + 2(-1 + \beta)(1 + 5\beta)m_q \left[(-1 + \psi)m_Q - \psi m_{Q'} \right] \right] \right. \\
 &\left. + \frac{3}{2} m_0^2 (1 - \beta)^2 \right\}, \tag{4.24}
 \end{aligned}$$

where,

$$\begin{aligned}
 \mu &= \frac{m_Q^2}{\psi} + \frac{m_{Q'}^2}{\eta} - s, \\
 \mu' &= \frac{m_Q^2}{\psi} + \frac{m_{Q'}^2}{1 - \psi} - s, \\
 \eta_{min} &= \frac{\psi m_{Q'}^2}{s\psi - m_Q^2}, \\
 \eta_{max} &= 1 - \psi, \\
 \psi_{min} &= \frac{1}{2s} \left[s + m_Q^2 - m_{Q'}^2 - \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right], \\
 \psi_{max} &= \frac{1}{2s} \left[s + m_Q^2 - m_{Q'}^2 + \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right]. \tag{4.25}
 \end{aligned}$$

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