PHYSICAL REVIEW D **87,** 016013 (2013)

Rare radiative $B_c \to D_{s1}(2460)\gamma$ transition in QCD

K. Azizi,[1,*] N. Ghahramani,[2,†] and A. R. Olamaei[2,‡]

[1]*Physics Department, Faculty of Arts and Sciences, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey*
[2]*Physics Department, College of Sciences, Shiraz University, Shiraz 71454, Iran*

(Received 6 July 2012; revised manuscript received 9 November 2012; published 22 January 2013)

We investigate the radiative $B_c \to D_{s1}\gamma$ transition in the framework of QCD sum rules. In particular, we calculate the transition form factors responsible for this decay in both weak annihilation and electromagnetic penguin channels using the quark condensate, mixed, and two-gluon condensate diagrams, as well as propagation of the soft quark in the electromagnetic field, as nonperturbative corrections. These form factors are then used to estimate the branching ratios of the channels under consideration. The total branching ratio of the $B_c \to D_{s1}\gamma$ transition is obtained to be of the order of 10^{-5}, and the dominant contribution comes from the weak annihilation channel.

DOI: 10.1103/PhysRevD.87.016013 PACS numbers: 11.55.Hx, 13.20.−v, 13.20.He

I. INTRODUCTION

The B_c is the only heavy meson consisting of two heavy quarks with different flavors; hence the decay properties of this meson are of special interest. The difference in heavy quark flavors forbids annihilation of this meson into gluons, so the excited B_c states undergo pionic or radiative transition to the pseudoscalar ground state when these states lie below the threshold of decay into the pair of heavy B and D mesons. The resulting pseudoscalar ground state is more stable compared to the corresponding quarkonia and decays mostly weakly. Because of this phenomenon, it is expected that experimental study of the B_c meson and its decay properties will constitute an important part of the physics program at LHCb. The study of the heavy mesons will not only provide a window into extracting the most accurate values of the Cabbibo-Kobayashi-Maskawa matrix elements as the sources of the CP violation in the Standard Model—it will also help us better understand the perturbative and nonperturbative aspects of QCD.

In the present study, we work out the rare radiative $B_c \to D_{s1}(2460)\gamma$ transition in the framework of the QCD sum rules [1,2]. Here $D_{s1}(2460)$ is the axial vector charmed-strange meson with quantum numbers $J^P = 1^+$ and interpolating current $\eta_\nu = \bar{s}\gamma_\nu\gamma_5 c$. This transition proceeds via both weak annihilation (WA) and electromagnetic penguin (EP) modes based on $b \to s\gamma$ at the quark level. We calculate the transition form factors responsible for this decay in both WA and EP modes using the quark condensate, mixed, and two-gluon condensate diagrams, as well as propagation of the soft quark in the electromagnetic field, as nonperturbative corrections. We then use these form factors to estimate the branching ratios in both modes, as well as the total branching fraction of the $B_c \to D_{s1}(2460)\gamma$ transition. As expected, the dominant contribution comes from the weak annihilation channel. Note that similar decays like the $B_c \to D_s^*\gamma$ transition have been studied in the same framework [3]. Some other radiative channels of the B_c meson like $B_c \to l\bar{\nu}\gamma$ and $B_c \to B_u^*\gamma$ have also been previously studied using the QCD sum rules technique [4,5]. For analysis of other decay channels of the B_c meson, see, for instance, Refs. [6–9].

The outline of the paper is as follows: In Sec. II, we consider the radiation of the photon from both B_c and D_{s1} mesons to construct the transition amplitude for the WA channel in terms of four relevant form factors. Two of the form factors [$F_V^{(B_c)}$ and $F_A^{(B_c)}$], responsible for the emission of the photon from the initial state, are calculated in Ref. [4], and the remaining two form factors [$F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$], representing the emission of the photon from the D_{s1} meson, are calculated in Sec. III. In Sec. IV, we consider the two gluon condensate contributions to calculate the transition form factors responsible for the EP mode. Finally, Sec. V is devoted to the numerical analysis of the form factors and the calculation of the decay rates and branching ratios for the modes under consideration. We also present results for the total decay rate and branching ratio of the $B_c \to D_{s1}(2460)\gamma$ transition. This section also contains our concluding remarks.

II. WEAK ANNIHILATION AMPLITUDE

In this section, we construct the WA amplitude for the radiative $B_c \to D_{s1}\gamma$ transition. Considering the quark contents of the initial and final mesonic states, the possible diagrams are shown in Fig. 1. Taking into account these diagrams, the transition amplitude for the radiative decay under consideration is written as

$$M^{\mathrm{WA}}(B_c \to D_{s1}\gamma) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \langle D_{s1}(p)\gamma(q)|(\bar{s}\Gamma_\nu c) \times (\bar{c}\Gamma^\nu b)|B_c(p+q)\rangle, \quad (1)$$

*kazizi@dogus.edu.tr
†ghahramany@susc.ac.ir
‡olamaei@gmail.com

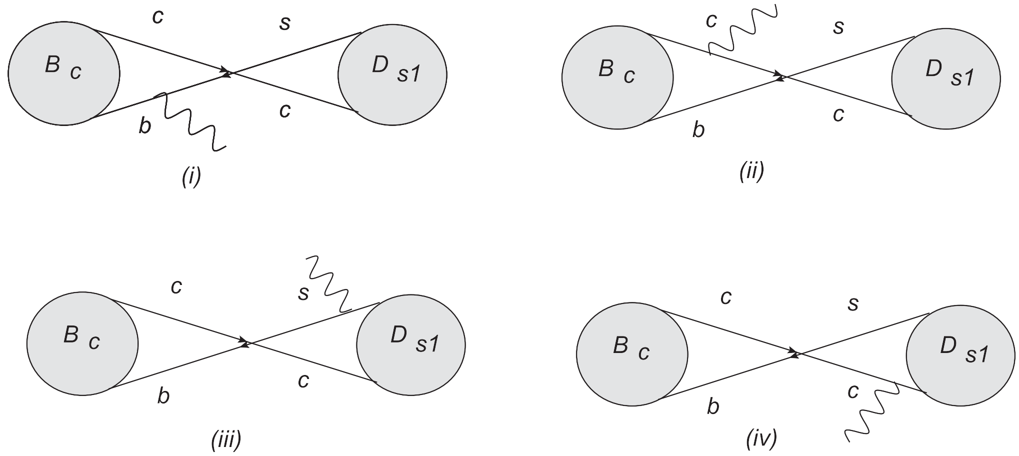

FIG. 1. The weak annihilation mechanism for $B_c \to D_{s1}\gamma$.

where G_F is the Fermi weak coupling constant; V_{ij} are elements of the Cabbibo-Kobayashi-Maskawa matrix; $\Gamma_\nu = \gamma_\nu(1-\gamma_5)$; and p, q, and $p+q$ are the momenta of the D_{s1} meson, photon, and B_c meson, respectively. To proceed further, we use the factorization hypothesis and write the transition matrix element in Eq. (1) as

$$\langle D_{s1}(p)\gamma(q)|(\bar{s}\Gamma_\nu c)(\bar{c}\Gamma^\nu b)|B_c(p+q)\rangle = -e\varepsilon^\mu\varepsilon^{(D_{s1})\nu} f_{D_{s1}} m_{D_{s1}} T^{(B_c)}_{\mu\nu} - ie\varepsilon^\mu(p+q)^\nu f_{B_c} T^{(D_{s1})}_{\mu\nu}, \quad (2)$$

where we have divided the matrix element into two separate parts: the emission of the photon from the B_c meson, represented by the covariant tensor $T^{(B_c)}_{\mu\nu}$ [diagrams (i) and (ii) in Fig. 1]; and the emission of the photon from the D_{s1} meson, denoted by the tensor $T^{(D_{s1})}_{\mu\nu}$ [diagrams (iii) and (iv) in Fig. 1]. In Eq. (2), f_{B_c} [$f_{D_{s1}}$] is the decay constant of the B_c [D_{s1}] meson, and ε^μ [$\varepsilon^{(D_{s1})\nu}$] is the polarization vector of the photon [D_{s1} meson]. The covariant tensors $T^{(B_c)}_{\mu\nu}$ and $T^{(D_{s1})}_{\mu\nu}$ are defined as

$$T^{(B_c)}_{\mu\nu}(p,q) \equiv i\int d^4x e^{iqx}\langle 0|T\{j^{\rm em}_\mu(x)\bar{c}(0)\Gamma_\nu b(0)\}|B_c(p+q)\rangle, \quad (3)$$

$$T^{(D_{s1})}_{\mu\nu}(p,q) \equiv i\int d^4x e^{iqx}\langle D_{s1}(p)|T\{j^{\rm em}_\mu(x)\bar{s}(0)\Gamma_\nu c(0)\}|0\rangle, \quad (4)$$

where $j^{\rm em}_\mu$ is the electromagnetic current and T is the time-ordering operator. By applying the Ward identity for the electromagnetic current and using $q^2 = 0$ for the real photon, $\varepsilon.q = 0$, and $\varepsilon^{(D_{s1})}.p = 0$—similar to what is done in Refs. [3,10,11]—we get the following results, corresponding to the emission of the photon from the initial and final mesonic states in terms of form factors:

$$e\varepsilon^\mu\varepsilon^{(D_{s1})\nu} f_{D_{s1}} m_{D_{s1}} T^{(B_c)}_{\mu\nu} = ef_{D_{s1}} m_{D_{s1}}\{[(\varepsilon.\varepsilon^{(D_{s1})})(p.q) - (\varepsilon.p)(\varepsilon^{(D_{s1})}.q)]iF^{(B_c)}_A + if_{B_c}(\varepsilon.\varepsilon^{(D_{s1})}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_{s1})\nu}\varepsilon^\mu p^\lambda q^\sigma F^{(B_c)}_V\}, \quad (5)$$

$$ie\varepsilon^\mu(p+q)^\nu f_{B_c} T^{(D_{s1})}_{\mu\nu} = ief_{B_c}\{[(\varepsilon.\varepsilon^{(D_{s1})})(p.q) - (\varepsilon.p)(\varepsilon^{(D_{s1})}.q)]iF^{(D_{s1})}_A + f_{D_{s1}} m_{D_{s1}}(\varepsilon.\varepsilon^{(D_{s1})}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_{s1})\nu}\varepsilon^\mu p^\lambda q^\sigma F^{(D_{s1})}_V\}, \quad (6)$$

where $F^{(B_c)}_{V(A)}$ and $F^{(D_{s1})}_{V(A)}$ are the transition form factors. Using Eqs. (5), (6), and (2), we find the WA transition amplitude to be

$$M^{\rm WA}(B_c \to D_{s1}\gamma) = e\frac{G_F}{\sqrt{2}} V_{cb}V^*_{cs}(-f_{D_{s1}} m_{D_{s1}}\{[(\varepsilon.\varepsilon^{(D_{s1})})(p.q) - (\varepsilon.p)(\varepsilon^{(D_{s1})}.q)]iF^{(B_c)}_A + if_{B_c}(\varepsilon.\varepsilon^{(D_{s1})}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_{s1})\nu}\varepsilon^\mu p^\lambda q^\sigma F^{(B_c)}_V\} - if_{B_c}\{[(\varepsilon.\varepsilon^{(D_{s1})})(p.q) - (\varepsilon.p)(\varepsilon^{(D_{s1})}.q)]iF^{(D_{s1})}_A + f_{D_{s1}} m_{D_{s1}}(\varepsilon.\varepsilon^{(D_{s1})}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_{s1})\nu}\varepsilon^\mu p^\lambda q^\sigma F^{(D_{s1})}_V\}). \quad (7)$$

As mentioned in Sec. I, the form factors $F^{(B_c)}_V$ and $F^{(B_c)}_A$ are calculated in Ref. [4], so what remain to be calculated are the form factors $F^{(D_{s1})}_V$ and $F^{(D_{s1})}_A$, which we discuss in the next section.

III. QCD SUM RULES FOR THE FORM FACTORS $F^{(D_{s1})}_V$ AND $F^{(D_{s1})}_A$

To calculate the transition form factors $F^{(D_{s1})}_V$ and $F^{(D_{s1})}_A$ via QCD sum rules formalism, we start considering the following correlation function:

$$\Pi_{\mu\nu}(p,q) = i\int d^4x e^{iQx}\langle\gamma(q)|T\{\bar{c}(x)\gamma_\mu(1-\gamma_5) \times s(x)\bar{s}(0)\gamma_\nu\gamma_5 c(0)\}|0\rangle, \quad (8)$$

where $Q = p + q$. The basic idea in this method is to calculate this correlation function, first in hadronic language (called the phenomenological or physical side), and second in terms of the QCD degrees of freedom, using the operator product expansion in deep Euclidean space (called the theoretical or QCD side). The two representations are then matched in order to get the QCD sum rules for the form factors. To suppress the contributions coming from the higher energy states and continuum, we apply a Borel transformation as well as continuum subtraction, which bring two auxiliary parameters: the Borel mass parameter and the continuum threshold. We shall find their working regions, requiring that the physical observables be independent of these parameters.

First, we focus on calculation of the phenomenological side. For this aim, we insert a full set of hadronic D_{s1} states into Eq. (8) and perform the four-integral over x to get

$$\Pi_{\mu\nu}(p,q) = \frac{\langle\gamma(q)|\bar{c}\gamma_\mu(1-\gamma_5)s|D_{s1}(p)\rangle\langle D_{s1}(p)|\bar{s}\gamma_\nu\gamma_5 c|0\rangle}{m_{D_{s1}}^2 - p^2}. \quad (9)$$

The matrix element $\langle D_{s1}(p)|\bar{s}\gamma_\nu\gamma_5 c|0\rangle$ is defined in terms of the decay constant and the polarization vector of the D_{s1} meson as

$$\langle D_{s1}(p)|\bar{s}\gamma_\nu\gamma_5 c|0\rangle = f_{D_{s1}} m_{D_{s1}} \varepsilon_\nu^{(D_{s1})}, \quad (10)$$

while the transition matrix element is parametrized in terms of form factors:

$$\langle\gamma(q)|\bar{c}\gamma_\mu(1-\gamma_5)s|D_{s1}(p)\rangle = e\bigg\{i\varepsilon_{\mu\alpha\beta\sigma}\varepsilon^\alpha\varepsilon^{(D_{s1})\beta}q^\sigma\frac{F_V^{(D_{s1})}(Q^2)}{m_{D_{s1}}^2} + [\varepsilon_\mu(\varepsilon^{(D_{s1})}.q) - (\varepsilon.\varepsilon^{(D_{s1})})q_\mu]\frac{F_A^{(D_{s1})}(Q^2)}{m_{(D_{s1})}^2}\bigg\}. \quad (11)$$

By substituting Eqs. (10) and (11) into Eq. (9) and summing over the polarization vector of the D_{s1} meson, we find the following result for the phenomenological part of the correlation function:

$$\Pi_{\mu\nu}(p,q) = \frac{ef_{D_{s1}}m_{D_{s1}}}{m_{D_{s1}}^2 - p^2}\bigg\{i\varepsilon_{\mu\nu\alpha\sigma}\varepsilon^\alpha q^\sigma\frac{F_V^{(D_{s1})}(Q^2)}{m_{D_{s1}}^2} + [q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu]\frac{F_A^{(D_{s1})}(Q^2)}{m_{D_{s1}}^2}\bigg\}. \quad (12)$$

We now compute the QCD side of the correlation function within the deep Euclidean region in terms of the QCD parameters. We start by writing the correlation function in terms of the two selected structures as

$$\Pi_{\mu\nu}(p,q) = i\varepsilon_{\mu\nu\alpha\sigma}\varepsilon^\alpha q^\sigma\Pi_1 + [q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu]\Pi_2, \quad (13)$$

where each function Π_i ($i = 1$ or 2) has perturbative and nonperturbative parts; i.e.,

$$\Pi_i = \Pi_i^{\rm pert} + \Pi_i^{\rm nonpert}. \quad (14)$$

To calculate the perturbative parts, we consider Figs. 2(a) and 2(b), where the photon can be radiated from both the charm and strange quarks. For the nonperturbative parts, we take into account the quark condensate and mixed diagrams [Figs. 2(c)–2(e)], as well as Fig. 2(f), for the interaction of the photon with the soft quark.

The perturbative part in each case can be written via the dispersion relation as

$$\Pi_i^{\rm pert} = \int ds\frac{\rho_i(s,Q^2)}{s-p^2} + \text{subtraction terms}, \quad (15)$$

where ρ_i are the spectral densities. Our main task is to calculate these spectral densities using the diagrams in Figs. 2(a) and 2(b). Here we use a method based on both Feynman and Schwinger parameterizations with several Borel transformations (see also Ref. [12]). The Feynman amplitude for the diagram in Fig. 2(a) can be written as

$$\Pi_{\mu\nu,(a)} = eN_cQ_s\int\frac{d^4k}{(2\pi)^4}\bigg\{\mathrm{Tr}\bigg[\frac{i(\not{k}+m_c)}{k^2+m_c^2}\gamma_\nu\gamma_5 \times \frac{i(\not{p}+\not{k}+m_s)}{(p+k)^2-m_s^2}\not{\varepsilon}\frac{i(\not{Q}+\not{k}+m_s)}{(Q+k)^2-m_s^2}\gamma_\mu(1-\gamma_5)\bigg]\bigg\}, \quad (16)$$

where $N_c = 3$ is the number of colors and Q_s is the charge of the strange quark. Using the Feynman parameterization,

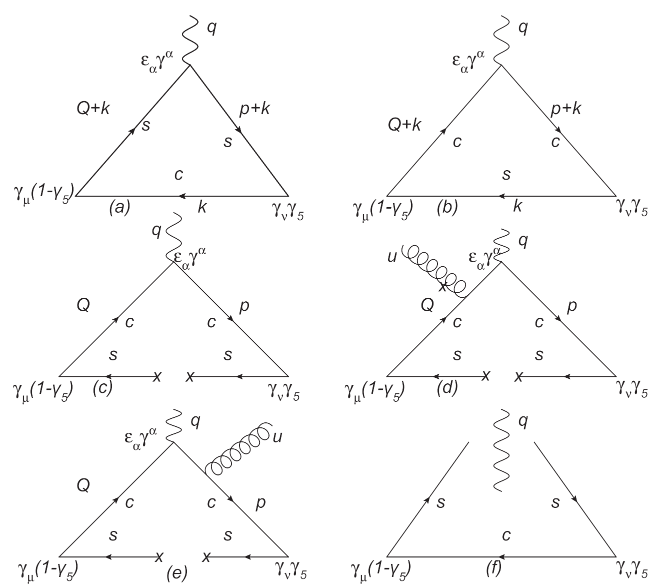

FIG. 2. Diagrams for bare-loop [(a), (b)], quark and mixed condensates [(c)–(e)] and propagation of the soft quark in the electromagnetic field (f).

we perform the four-integral over k, and then use the Schwinger parameterization

$$\frac{1}{\Delta^n}=\frac{1}{\Gamma(n)}\int_0^\infty d\alpha\alpha^{n-1}e^{-\alpha\Delta} \tag{17}$$

to write the denominators in exponential forms. As a result, we get

$$\Pi_{1,a}^{\text{pert}}=\frac{eN_cQ_s}{4\pi^2}\bigg\{\int_0^1 dxx\int_0^1 dy[m_s(m_c+m_sxy)+p^2x\bar{x}(1-\bar{x}y)+2p.q\bar{x}x^2y^2]\int_0^\infty d\alpha e^{-\alpha\Delta}\bigg\}, \tag{18}$$

$$\Pi_{2,a}^{\text{pert}}=\frac{eNcQ_s}{4\pi^2}\bigg\{\int_0^1 dxx\int_0^1 dyx[m_s(m_c-m_sxy)-p^2x\bar{x}(1-\bar{x}y)-2p.q\bar{x}x^2y^2]\int_0^\infty d\alpha e^{-\alpha\Delta}\bigg\}, \tag{19}$$

where $\bar{x}(\bar{y})=1-x(y)$ and $\Delta=m_c^2\bar{x}+m_s^2x-p^2x\bar{x}\,\bar{y}-Q^2x\bar{x}y$.

By applying a double Borel transformation $\hat{B}(M_1^2)\hat{B}(M_2^2)$ on Π_i^{pert} that transforms $Q^2\to M_1^2$ and $p^2\to M_2^2$, we obtain

$$\hat{\Pi}_{1,a}^{\text{pert}}=\frac{eN_cQ_s}{4\pi^2}\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}\int_0^1 dx\frac{1}{\bar{x}}e^{\frac{(m_c^2\bar{x}+m_s^2x)(\sigma_1+\sigma_2)}{\bar{x}x}}\times\bigg\{m_cm_s+m_s^2x\frac{\sigma_1}{\sigma_1+\sigma_2}+2x(1-x^2)\frac{\sigma_1}{(\sigma_1+\sigma_2)^2}\bigg\}, \tag{20}$$

$$\hat{\Pi}_{2,a}^{\text{pert}}=\frac{eN_cQ_s}{4\pi^2}\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}\int_0^1 dx\frac{1}{\bar{x}}e^{\frac{(m_c^2\bar{x}+m_s^2x)(\sigma_1+\sigma_2)}{\bar{x}x}}\times\bigg\{m_cm_s-m_s^2x\frac{\sigma_1}{\sigma_1+\sigma_2}-2x(1-x^2)\frac{\sigma_1}{(\sigma_1+\sigma_2)^2}\bigg\}, \tag{21}$$

where $\sigma_{1,2}=1/M_{1,2}^2$, and we have used

$$\begin{aligned}\hat{B}_{p^2}(M^2)e^{-\alpha p^2}&=\delta(1-\alpha M^2),\\ \hat{B}_{p^2}(M^2)p^2e^{-\alpha p^2}&=-\frac{d}{d\alpha}\\ \hat{B}_{p^2}(M^2)e^{-\alpha p^2}&=-\frac{d}{d\alpha}\delta(1-\alpha M^2).\end{aligned} \tag{22}$$

Now, we perform a second double Borel transformation on $\hat{\Pi}_i^{\text{pert}}$ in order to transform σ_1 and σ_2 to the new variables w and s using

$$\varrho_i(w,s)=\frac{1}{ws}\hat{B}\Big(\frac{1}{w},\sigma_1\Big)\hat{B}\Big(\frac{1}{s},\sigma_2\Big)\frac{\hat{\Pi}_i^{\text{pert}}}{\sigma_1\sigma_2}. \tag{23}$$

In our calculations, we also use the relations

$$\hat{B}\Big(\frac{1}{w},\sigma_1\Big)\hat{B}\Big(\frac{1}{s},\sigma_2\Big)e^{-\alpha(\sigma_1+\sigma_2)}=\delta\Big(1-\frac{\alpha}{w}\Big)\delta\Big(1-\frac{\alpha}{s}\Big) \tag{24}$$

and

$$\sigma^ne^{-\alpha\sigma}=\Big(-\frac{d}{d\alpha}\Big)^ne^{-\alpha\sigma}. \tag{25}$$

The final expressions for the spectral densities are then calculated via the following formula:

$$\rho_i(s,Q^2)=\int dw\frac{\varrho_i(w,s)}{w-Q^2}. \tag{26}$$

After lengthy calculations, we get the following spectral densities corresponding to Fig. 2(a):

$$\begin{aligned}\rho_{1a}(s,Q^2)=\frac{eN_cQ_s}{16\pi^2}\frac{1}{(s-Q^2)^2}\int_{x_0}^{x_1}dx\frac{1}{x\bar{x}^2}\{&m_c^4\bar{x}^2(x-5)\\&+m_s^4x^2(x-6)-m_c^2m_s^2x\bar{x}(2x-11)\\&+4m_cm_sx\bar{x}(s-Q^2)+m_c^2x\bar{x}^2[(\bar{x}-8)Q^2\\&+\bar{x}(s-Q^2)]-m_s^2x^2\bar{x}[(x-10)Q^2\\&+(x-2)(s-Q^2)]-4x^2\bar{x}^2Q^2(s-Q^2)\\&-4x^2\bar{x}^2Q^4\},\end{aligned} \tag{27}$$

$$\begin{aligned}\rho_{2a}(s,Q^2)=\frac{eN_cQ_s}{16\pi^2}\frac{1}{(s-Q^2)^2}\int_{x_0}^{x_1}dx\frac{1}{x\bar{x}^2}\{&-m_c^4\bar{x}^2(x-5)\\&-m_s^4x^2(-6+x)+m_c^2m_s^2x\bar{x}(2x-11)\\&+4m_cm_sx\bar{x}(s-Q^2)-m_c^2x\bar{x}^2[(\bar{x}-8)Q^2\\&+\bar{x}(s-Q^2)]+m_s^2x^2\bar{x}[(x-10)Q^2\\&+(x-2)(s-Q^2)]+4x^2\bar{x}^2Q^2(s-Q^2)\\&+4x^2\bar{x}^2Q^4\},\end{aligned} \tag{28}$$

where the integral boundaries x_0 and x_1 satisfy the following inequality:

$$sx\bar{x}-(m_c^2\bar{x}+m_s^2x)\geq 0, \tag{29}$$

which comes from the Heaviside theta function arising in these calculations. Similarly, we calculate the contribution of Fig. 2(b). The final expressions for the spectral densities corresponding to the two selected structures are

$$\rho_1(s,Q^2) = \frac{eN_c}{32\pi^2}\frac{1}{(s-Q^2)^2}\Big\{Q_s\Big[\lambda(4(5\alpha-5\beta-1)sQ^2+s^2[3\alpha(\alpha-3)-\beta(6\alpha+1)+3\beta^2])+2(4m_cm_s(s-Q^2)+8\alpha sQ^2+\alpha(1-4\alpha+9\beta)s^2)\ln\Big(\frac{1+\alpha-\beta-\lambda}{1+\alpha-\beta+\lambda}\Big)\Big]+Q_c\Big[\lambda(4(5\beta-5\alpha-1)sQ^2+s^2[3\beta(\beta-3)-\alpha(6\beta+1)+3\alpha^2])+2(4m_cm_s(s-Q^2)+8\beta sQ^2+\beta(1-4\beta+9\alpha)s^2)\ln\Big(\frac{1+\beta-\alpha-\lambda}{1+\beta-\alpha+\lambda}\Big)\Big]\Big\}, \quad (30)$$

$$\rho_2(s,Q^2) = \frac{eN_c}{32\pi^2}\frac{1}{(s-Q^2)^2}\Big\{Q_s\Big[\lambda(4(-5\alpha+5\beta+1)sQ^2+s^2[-3\alpha(\alpha-3)+\beta(6\alpha+1)-3\beta^2])+2(4m_cm_s(s-Q^2)-8s\alpha Q^2-\alpha(1-4\alpha+9\beta)s^2)\ln\Big(\frac{1+\alpha-\beta-\lambda}{1+\alpha-\beta+\lambda}\Big)\Big]+Q_c\Big[\lambda(4(-5\beta+5\alpha+1)sQ^2+s^2[-3\beta(\beta-3)+\alpha(6\beta+1)-3\alpha^2])+2(4m_cm_s(s-Q^2)-8s\beta Q^2-\beta(1-4\beta+9\alpha)s^2)\ln\Big(\frac{1+\beta-\alpha-\lambda}{1+\beta-\alpha+\lambda}\Big)\Big]\Big\}, \quad (31)$$

where $\alpha=\frac{m_s^2}{s}$, $\beta=\frac{m_c^2}{s}$, and $\lambda=\sqrt{1+\alpha^2+\beta^2-2\alpha-2\beta-2\alpha\beta}$.

For the nonperturbative parts, we first consider the quark condensate and mixed diagrams [Figs. 2(c)–2(e)]. For their contributions, we get

$$\Pi^{\text{nonpert}}_{1(c,d,e)} = \frac{m_c}{r^2R^2}\langle\bar ss\rangle+\frac{m_s}{2}\langle\bar ss\rangle\Big[\frac{2}{r^2R^2}+\frac{m_c^2}{r^4R^2}-\frac{7m_c^2}{r^2R^4}-\frac{4m_c^4}{r^4R^4}\Big]+\frac{m_s^2}{2}\langle\bar ss\rangle\Big[\frac{2m_c^3}{r^2R^6}-\frac{8m_c^5}{r^6R^4}+\frac{2m_c^3}{r^4R^4}-\frac{3m_c}{r^2R^4}+\frac{2m_c^3}{r^6R^2}-\frac{m_c}{r^4R^2}\Big]+\frac{m_0^2}{12}\langle\bar ss\rangle\Big[\frac{-6m_c^3}{r^2R^6}+\frac{24m_c^5}{r^6R^4}-\frac{6m_c^3}{r^4R^4}+\frac{8m_c}{r^2R^4}-\frac{6m_c^3}{r^6R^2}+\frac{3m_c}{r^4R^2}\Big]-\frac{4m_c^3}{r^2R^4}\langle\bar ss\rangle, \quad (32)$$

$$\Pi^{\text{nonpert}}_{2(c,d,e)} = -\frac{m_c}{r^2R^2}\langle\bar ss\rangle+\frac{m_s}{2}\langle\bar ss\rangle\Big[\frac{1}{r^2R^2}+\frac{1}{R^4}-\frac{4m_c^4}{r^4R^4}-\frac{3m_c^2}{r^2R^4}+\frac{m_c^2}{2r^4R^2}\Big]+\frac{m_s^2}{2}\langle\bar ss\rangle\Big[-\frac{2m_c^3}{r^2R^6}+\frac{8m_c^5}{r^6R^4}-\frac{2m_c^3}{r^4R^4}+\frac{3m_c}{r^2R^4}-\frac{2m_c^3}{r^6R^2}+\frac{m_c}{r^4R^2}\Big]+\frac{m_0^2}{4}\langle\bar ss\rangle\Big[\frac{2m_c^3}{r^2R^6}-\frac{8m_c^5}{r^6R^4}+\frac{2m_c^3}{r^4R^4}-\frac{4m_c}{r^2R^4}+\frac{2m_c^3}{r^6R^2}-\frac{m_c}{r^4R^2}\Big]+\frac{4m_c^3}{r^2R^4}\langle\bar ss\rangle, \quad (33)$$

where $r^2=p^2-m_c^2$ and $R^2=Q^2-m_c^2$.

The final contribution to the WA mode is that of Fig. 2(f). This diagram corresponds to the propagation of the soft quark in the external electromagnetic field. Here we need to make use of the light cone version of the QCD sum rules and photon distribution amplitudes (DAs). The relevant correlation function is of the form

$$\Pi_{\mu\nu,(f)}(p,q) = i\int d^4x e^{-iQx}\langle\gamma(q)|T\{\bar s(0)\gamma_\mu\gamma_5 c(0)\bar c(x)\times\gamma_\nu(1-\gamma_5)s(x)\}|0\rangle. \quad (34)$$

By contracting the c-quark lines in Eq. (34) and using the propagator of the heavy quark in momentum space, we obtain

$$\Pi_{\mu\nu,(f)}(p,q) = i^2\int d^4x\frac{d^4k}{(2\pi)^4}\frac{e^{-i(Q-k)x}}{m_c^2-k^2}\times\langle\gamma(q)|\bar s\gamma_\mu\gamma_5(\not k+m_c)\gamma_\nu(1-\gamma_5)s|0\rangle. \quad (35)$$

To relate the matrix element in the above equation to the photon DAs, we use the identities

$$\gamma_\mu\gamma_\nu = g_{\mu\nu}+i\sigma_{\mu\nu},$$
$$\gamma_\mu\gamma_\nu\gamma_5 = g_{\mu\nu}\gamma_5-\frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma_{\alpha\beta},$$
$$\gamma_\mu\gamma_\alpha\gamma_\nu = g_{\mu\alpha}\gamma_\nu+g_{\nu\alpha}\gamma_\mu-g_{\mu\nu}\gamma_\alpha+i\varepsilon_{\mu\nu\alpha\lambda}\gamma_\lambda\gamma_5. \quad (36)$$

The relevant photon DAs of twist 2, 3, and 4 (Refs. [13,14]) are

$$\langle\gamma(q)|\bar s\gamma_\nu s|0\rangle = -\frac{Q_s}{2}f_{3\gamma}\int_0^1 du\bar\psi(u)x^\theta F_{\theta\nu}(ux),$$
$$\langle\gamma(q)|\bar s\gamma_\alpha\gamma_5 s|0\rangle = -\frac{iQ_s}{4}f_{3\gamma}\int_0^1 du\bar\psi^{(A)}(u)x^\theta\tilde F_{\theta\alpha}(ux),$$
$$\langle\gamma(q)|\bar s\sigma_{\alpha\beta}s|0\rangle = Q_s\langle\bar ss\rangle\int_0^1 du\phi(u)F_{\alpha\beta}(ux)+\frac{Q_s\langle\bar ss\rangle}{16}\int_0^1 dux^2\mathbf{A}(u)F_{\alpha\beta}(ux)+\frac{Q_s\langle\bar ss\rangle}{8}\int_0^1 du\mathbf{B}(u)\times x^\rho(x_\beta F_{\alpha\rho}(ux)-x_\alpha F_{\beta\rho}), \quad (37)$$

where $F_{\mu\nu}$ is the field strength tensor of the electromagnetic field, defined by

$$F_{\mu\nu}(x) = -i(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)e^{iqx} \tag{38}$$

and

$$\tilde{F}_{\mu\nu}(x) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}(x). \tag{39}$$

The wave function $\phi(u)$ is defined in terms of the magnetic susceptibility $\chi(\mu)$ at a renormalization scale ($\mu = 1$ GeV^2) in the following manner:

$$\phi(u) = \chi(\mu)u(1-u). \tag{40}$$

The remaining functions, $\bar{\psi}^{(V)}(u)$, $\bar{\psi}^{(A)}(u)$, $\mathbf{A}(u)$, and $\mathbf{B}(u)$, are also defined as [13,14]

$$\begin{aligned}
\bar{\psi}^{(V)}(u) &= -20u(1-u)(2u-1) + \frac{15}{16}(\omega_\gamma^A - 3\omega_\gamma^V)u(1-u)(2u-1)(7(2u-1)^2-3),\\
\bar{\psi}^{(A)}(u) &= (1-(2u-1)^2)(5(2u-1)^2-1)\frac{5}{2}\left(1+\frac{19}{16}\omega_\gamma^V - \frac{3}{16}\omega_\gamma^A\right),\\
\mathbf{A}(u) &= 40u(1-u)(3k-k^+ +1) + 8(\xi_2^+ - 3\xi_2)[u(1-u)(2+13u(1-u)) + 2u^3(10-15u+16u^2)\ln u\\
&\quad + 2(1-u)^3(10-15(1-u)+6(1-u^2))\ln(1-u)],\\
\mathbf{B}(u) &= 40\int_0^u d\alpha(4-\alpha)(1+3k^+)\left[-\frac{1}{2}+\frac{3}{2}(2\alpha-1)^2\right],
\end{aligned} \tag{41}$$

where k, k^+, ξ_2, ξ_2^+, and $f_{3\gamma}$ are constants (see Refs. [13,14]). After combining the above equations, we perform the four-integrals over x and k. The coefficients of the corresponding structures $i\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\alpha q^\beta$ and $[q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu]$ are obtained as follows:

$$\begin{aligned}
\Pi_{1f}^{\rm nonpert}(p,q) &= \frac{Q_s}{2(m_c^2-p^2)^3}\int_0^1 du\{m_c^3\langle\bar{s}s\rangle\mathbf{A}(u)\\
&\quad + (m_c^2-p^2)[m_c\langle\bar{s}s\rangle\mathbf{B}(u) - 2(5m_c^2-p^2)\\
&\quad \times (m_c\langle\bar{s}s\rangle\phi(u) - f_{3\gamma}\psi^{(V)}(u))]\},
\end{aligned} \tag{42}$$

$$\begin{aligned}
\Pi_{2f}^{\rm nonpert}(p,q) &= \frac{m_cQ_s}{2(m_c^2-p^2)^3}\int_0^1 du\{\mathbf{A}(u)m_c^2\langle\bar{s}s\rangle\\
&\quad + 2(-5m_c^2+p^2)\langle\bar{s}s\rangle\phi(u) + (m_c^2-p^2)\\
&\quad \times [\mathbf{B}(u)\langle\bar{s}s\rangle + f_{3\gamma}m_c\psi^{(A)}(u)]\}.
\end{aligned} \tag{43}$$

Now, to find the QCD sum rules for the form factors, we match the coefficients of the selected structures from both the phenomenological and QCD sides and perform the Borel transformation with respect to the momentum of the D_{s1} meson ($p^2 \to M_B^2$). To further suppress the contributions of the higher energy states and continuum, we also perform the continuum subtraction and use the quark-hadron duality assumption. As a result, we find

$$\begin{aligned}
F_{V,A}^{(D_{s1})}(Q^2) &= \frac{m_{D_{s1}}}{f_{D_{s1}}}e^{m_{D_{s1}}/M_B^2}\hat{B}\left\{\int_{(m_s+m_c)^2}^{s_0} ds\frac{\rho_{1,2}(s,Q^2)}{s-p^2}\right.\\
&\quad \left. + \Pi_{1,2(c+d+e+f)}^{\rm nonpert}\right\},
\end{aligned} \tag{44}$$

where s_0 is the continuum threshold, and the V (A) on the left-hand side corresponds to the 1 (2) on the right-hand side. To obtain the expressions for the above sum rules in the Borel scheme, we perform the Borel transformation using the standard rule

$$\hat{B}\frac{1}{(p^2-s)^n} = (-1)^n\frac{e^{-s/M_B^2}}{\Gamma(n)(M_B^2)^{n-1}}. \tag{45}$$

IV. QCD SUM RULES FOR THE FORM FACTORS RESPONSIBLE FOR THE ELECTROMAGNETIC PENGUIN MODE

At the quark level, the EP transition of the $B_c \to D_{s1}\gamma$ proceeds via $b \to s\gamma$, whose effective Hamiltonian is written as

$$\begin{aligned}
H^{\rm eff} &= -\frac{G_Fe}{4\pi^2\sqrt{2}}V_{tb}V_{ts}^*C_7(\mu)\bar{s}\sigma_{\mu\nu}\left[m_b\frac{1+\gamma_5}{2}\right.\\
&\quad \left. + m_s\frac{1-\gamma_5}{2}\right]bF^{\mu\nu}.
\end{aligned} \tag{46}$$

The amplitude of this mode is obtained from

$$M^{\rm EP} = \langle D_{s1}(p)|H^{\rm eff}|B_c(Q)\rangle; \tag{47}$$

hence to proceed further, we need to calculate the following matrix elements:

$$\langle D_{s1}|\bar{s}\sigma_{\mu\nu}(1\pm\gamma_5)q^\nu b|B_c\rangle, \tag{48}$$

which can be parametrized in terms of two gauge-invariant form factors, $T_1(q^2)$ and $T_2(q^2)$, in the case of a real photon, i.e.,

$$\begin{aligned}
&\langle D_{s1}(p,\varepsilon^{(D_{s1})})|\bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b|B_c(Q)\rangle\\
&\quad = i\varepsilon_{\mu\alpha\beta\lambda}\varepsilon^{(D_{s1})\alpha}p^\beta Q^\lambda T_1(0),\\
&\langle D_{s1}(p,\varepsilon^{(D_{s1})})|\bar{s}\sigma_{\mu\nu}q^\nu b|B_c(Q)\rangle\\
&\quad = [(m_{B_c}^2 - m_{D_{s1}}^2)\varepsilon_\mu^{(D_{s1})} - (\varepsilon^{(D_{s1})}.q)(p+Q)_\mu]T_2(0),
\end{aligned} \tag{49}$$

where these two form factors are not independent from each other. Using the relation $\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$, we see $T_1(0) = \frac{1}{2}T_2(0)$. Therefore, we need to calculate just one of them, and here we choose to calculate the form factor $T_2(0)$. The corresponding correlation function is chosen as

$$\Pi_{\mu\nu}(p^2, Q^2) = i^2 \iint d^4x d^4y e^{-i(Qx-py)} \langle 0|T\{\bar{c}(y)\gamma_\nu\gamma_5 s(y) \times \bar{s}(0)\sigma_{\mu\alpha}q^\alpha b(0)\bar{b}(x)\gamma_5 c(x)\}|0\rangle, \quad (50)$$

where $\bar{b}\gamma_5 c$ and $\bar{c}\gamma_\nu\gamma_5 s$ are the interpolating currents of the initial and final mesonic states, respectively. Here also, $\bar{s}\sigma_{\mu\alpha}q^\alpha b$ is the transition current. Using the general philosophy of the QCD sum rules, we calculate this correlation function again in two different languages: the hadronic language and the quark-gluon language. For the hadronic or phenomenological side, we get

$$\Pi_{\mu\nu}(p^2, Q^2) = \frac{i f_{D_{s1}} f_{B_c} m_{D_{s1}} m_{B_c}^2}{(m_{B_c}^2 - Q^2)(m_{D_{s1}}^2 - p^2)(m_b + m_c)} \times \Big\{(m_{B_c}^2 - m_{D_{s1}}^2) g_{\mu\nu} T_2(0) - \Big(\frac{m_{B_c^2} - m_{D_{s1}}^2}{m_{D_{s1}}^2}\Big) p_\mu p_\nu T_2(0) + (p+Q)_\mu \times \Big[\frac{p.q}{m_{D_{s1}}^2} p_\nu - q_\nu\Big] T_2(0)\Big\} + \cdots, \quad (51)$$

where the ellipsis denotes contributions of the higher energy states and continuum which will be suppressed by applying the Borel transformation as well as the continuum subtraction. In deriving the above equation, we have used the following definition of the decay constant of the B_c meson:

$$\langle B_c|\bar{b}\gamma_5 c|0\rangle = i\frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)}. \quad (52)$$

Note that to calculate the form factor $T_2(0)$, we choose the structure $g_{\mu\nu}$.

On the QCD side, the correlation function is written in terms of the selected structure as

$$\Pi_{\mu\nu} = g_{\mu\nu}\Pi(p^2, Q^2), \quad (53)$$

where

$$\Pi(p^2, Q^2) = \Pi^{\mathrm{pert}}(p^2, Q^2) + \Pi^{\mathrm{nonpert}}(p^2, Q^2). \quad (54)$$

Here the perturbative part is related to the spectral density $\rho^{\mathrm{pert}}(s', t)$ by a double dispersion integral,

$$\Pi^{\mathrm{pert}}(p^2, Q^2) = -\frac{1}{(2\pi)^2}\iint ds' dt \frac{\rho^{\mathrm{pert}}(s', t)}{(s' - Q^2)(t - p^2)} + \text{subtraction terms}, \quad (55)$$

and for the nonperturbative contributions we will calculate the two-gluon condensate diagrams.

Now, we focus our attention on calculating the spectral density. Using the Cutkosky method [15], we get

$$\rho^{\mathrm{per}}(s', t) = 2Nc\{I_0[\Delta[(m_b - m_c)(m_c + m_s) + t] - \Delta'[(m_b - m_c)(m_c + m_s) + s'] + 2m_c[(m_c + m_s)s' + m_b t - m_c t] - m_c(m_b + m_s)u] + 2A1(-2s' + u)\}, \quad (56)$$

where

$$I_0 = \frac{1}{4\sqrt{\lambda'(s', t, q^2)}},$$
$$\lambda'(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc,$$
$$A_1 = \frac{-I_0}{(-4s't + u^2)^2}[\Delta^2 t + \Delta'^2 s' - \Delta\Delta' u + m_c^2(-4s't + u^2)], \quad (57)$$
$$\Delta = s' + m_c^2 - m_b^2,$$
$$\Delta' = t + m_c^2 - m_s^2,$$
$$u = t + s' - q^2.$$

Note that, to obtain the above spectral density, we have performed the integrals over the delta functions, which restricts the boundaries of the integrals over s' and t:

$$m_c^2 \le t \le t_0, \qquad t - \frac{t m_b^2}{m_c^2 - t} \le s' \le s_0', \quad (58)$$

where s_0' and t_0 are the continuum thresholds in the initial and final channels in the case of EP mode. There are several sources for nonperturbative contributions, such as quark-quark, quark-gluon and gluon-gluon condensates. However, the quark-quark and quark-gluon condensates give zero contributions after applying the double Borel transformation with respect to Q^2 ($Q^2 \to M_1^2$) and p^2 ($p^2 \to M_2^2$). Therefore, the remaining source of the nonperturbative contributions would be the gluon condensates (see Fig. 3). The calculation of such contributions is lengthy but standard. For the nonperturbative part in the Borel scheme, we get

$$\Pi^{\mathrm{nonpert}} = M_1^2 M_2^2 \Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle C_{G^2}, \quad (59)$$

where C_{G^2} is the Wilson coefficient of the gluon condensates, defined as

$$C_{G^2} = C_{G^2}^a + C_{G^2}^b + C_{G^2}^c + C_{G^2}^d + C_{G^2}^e + C_{G^2}^f. \quad (60)$$

The explicit expressions of $C_{G^2}^i$ are given in the Appendix.

Using a similar procedure to that presented in the previous section, we find the sum rule for the form factor $T_2(0)$ to be

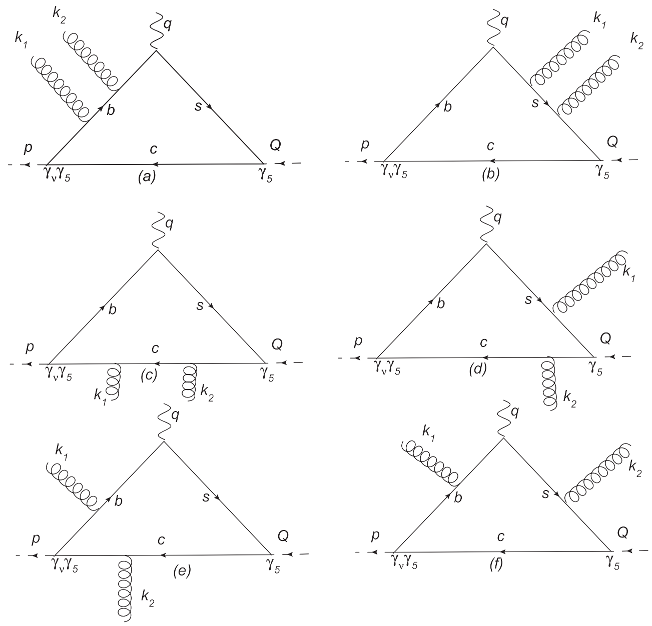

FIG. 3. Feynman diagrams for gluon condensate corrections.

$$T_2(0) = \frac{(m_b + m_c)e^{m_{B_c}^2/M_1^2}e^{m_{D_{s1}}^2/M_2^2}}{if_{D_{s1}}f_{B_c}m_{D_{s1}}m_{B_c}^2(m_{B_c}^2 - m_{D_{s1}}^2)} \times \left[\frac{-1}{(2\pi)^2}\iint ds' dt e^{-s'/M_1^2}e^{-t/M_2^2}\rho^{\rm pert}(s', t) + M_1^2 M_2^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle C_{G^2}\right]. \quad (61)$$

V. NUMERICAL ANALYSIS

This section is devoted to the numerical analysis of the form factors, estimating the branching ratio in the WA and EP channels and the total branching fraction of the $B_c \to D_{s1}(2460)\gamma$ transition. For this aim, we use the following quark and meson masses: $m_c = (1.275 \pm 0.015)$ GeV, $m_s \simeq 142$ MeV [16], $m_b = (4.7 \pm 0.1)$ GeV [17], $m_{D_{s1}} = (2459.6 \pm 0.6)$ MeV, and $m_{B_c} = (6.277 \pm 0.006)$ GeV [18]. For the values of the decay constants, we use $f_{D_{s1}} = (225 \pm 25)$ MeV and $f_{B_c} = (350 \pm 25)$ MeV [19–21]. The values of the condensates are as follows [17]: $\langle\bar\psi\psi|_{\mu=1\,{\rm GeV}}\rangle = -(240 \pm 10\,{\rm MeV})^3$, $\langle\bar s s\rangle = (0.8 \pm 0.2) \times \langle\bar\psi\psi\rangle$, $m_0^2 = (0.8 \pm 0.2)$ GeV^2, and $\langle\frac{\alpha_s}{\pi}G^2\rangle = (0.012 \pm 0.004)$ GeV^4. The parameters entered the photon DAs are also taken as $\chi = (3.15 \pm 0.30)$ GeV^{-2}, $k = 0.2$, $k^+ = 0$, $\zeta_1 = 0.4$, $\zeta_1^+ = 0$, $\zeta_2 = 0.3$, $\zeta_2^+ = 0$, $f_{3\gamma} = -(4 \pm 2) \times 10^{-3}$ GeV^2, $\omega_\gamma^A = -2.1 \pm 1.0$, and $\omega_\gamma^V = 3.8 \pm 1.8$ [13,14,22]. The remaining parameters are chosen as $|V_{cs}| = 0.957 \pm 0.017$, $|V_{cb}| = 0.0416 \pm 0.0006$, $|V_{tb}| = 0.77^{+0.18}_{-0.24}$, $|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}$ [18], $C_7(\mu = m_c) = -0.0068 - 0.02i$ [23], and $\tau_{B_c} = 0.52 \times 10^{-12}$ s.

The sum rules for the form factors also contain the continuum thresholds and the Borel mass parameters as auxiliary objects. We find the working regions for these parameters such that the physical observables are practically independent of them. The continuum thresholds are not completely arbitrary but are correlated with the energy of the first excited states in the initial and final mesonic channels. Our numerical results show that the results depend weakly on the thresholds in the intervals $s_0 = t_0 = (6–8)$ GeV^2 and $s'_0 = (45–50)$ GeV^2. The working regions for the Borel parameters are obtained by demanding not only that the contributions of the higher states and continuum be effectively suppressed, but also that the contributions of the higher-order operators and higher-twist DAs remain small; i.e., the series of sum rules must converge. These conditions lead to the intervals 6 GeV$^2 \le M_B^2 \le$ 12 GeV^2, 10 GeV$^2 \le M_1^2 \le$ 30 GeV^2, and 5 GeV$^2 \le M_2^2 \le$ 12 GeV^2 for the Borel mass parameters.

Now, we proceed to find the fit functions of the form factors using the aforesaid working regions for the auxiliary parameters. Here we would like to mention that to calculate the decay rates, we need only the values of the form factors $F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$ at $Q^2 = m_{B_c}^2$, $F_V^{(B_c)}$ and $F_A^{(B_c)}$ at $p^2 = m_{D_{s1}}^2$, and T_2 at $q^2 = 0$. However, we determine their fit functions in general and give their values at these fixed points. The fit functions for the form factors $F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$ are

$$f(Q^2) = \frac{f(0)}{1 + a\frac{Q^2}{m_{D_{s1}}^2} + b(\frac{Q^2}{m_{D_{s1}}^2})^2}, \quad (62)$$

where $f(0)$, a, and b are the fit parameters whose values are given in Table I.

The values of these form factors at $Q^2 = m_{B_c}^2$ are

$$F_V^{(D_{s1})}(Q^2 = m_{B_c}^2) = 0.055 \pm 0.016,$$
$$F_A^{(D_{s1})}(Q^2 = m_{B_c}^2) = -0.102 \pm 0.030, \quad (63)$$

where the errors on the values are due to the uncertainties in the determination of the working regions for the auxiliary parameters, as well as those coming from the DAs and other input parameters.

TABLE I. Fit parameters for the form factors $F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$.

Form factors	$f(0)$	a	b
$F_V^{(D_{s1})}(Q^2)$	0.098	0.171	−0.008
$F_A^{(D_{s1})}(Q^2)$	−2.478	3.644	−0.005

The fit functions for the form factors $F_{A,V}^{(B_c)}$ are [4]

$$F_V^{(B_c)}(p^2) = \frac{F_V(0)}{1 - p^2/m_1^2}, \qquad F_A^{(B_c)}(p^2) = \frac{F_A(0)}{1 - p^2/m_2^2}, \tag{64}$$

where the fit parameters are

$$F_V(0) = 0.44\ \text{GeV}, \qquad m_1^2 = 43.10\ \text{GeV}^2,$$
$$F_A(0) = 0.21\ \text{GeV}, \qquad m_2^2 = 48.00\ \text{GeV}^2.$$

The values of the form factors $F_{V,A}^{(B_c)}$ calculated at $p^2 = m_{D_{s1}}^2$ are

$$F_V^{(B_c)}(p^2 = m_{D_{s1}}^2) = (0.51 \pm 0.14)\ \text{GeV},$$
$$F_A^{(B_c)}(p^2 = m_{D_{s1}}^2) = (0.24 \pm 0.07)\ \text{GeV}. \tag{65}$$

For the form factor induced by the EP at $q^2 = 0$, we obtain

$$T_2(0) = -0.298 \pm 0.085. \tag{66}$$

At the end of this section, we would like to calculate the decay widths and branching ratios. Using the amplitudes of each decay mode, we find the following expressions for the decay rates at fixed points in WA and EP channels, as well as for the total decay rate of the transition under consideration:

$$\Gamma^{(\mathrm{WA})}(B_c \to D_{s1}\gamma) = \frac{G_F^2 \alpha |V_{cb}V_{cs}^*|^2}{16}\left(\frac{m_{B_c}^2 - m_{D_{s1}}^2}{m_{B_c}}\right)^3 \times \left\{ f_{B_c}^2 [(F_A^{(D_{s1})})^2 + (F_V^{(D_{s1})})^2] + 2 f_{B_c} f_{D_{s1}} F_V^{(B_c)} F_V^{(D_{s1})} \frac{m_{D_{s1}}}{m_{B_c}^2} + f_{D_{s1}}^2 m_{D_{s1}}^2 \left[\frac{(F_A^{(B_c)})^2}{m_{B_c}^4} + \frac{(F_V^{(B_c)})^2}{m_{B_c}^4}\right]\right\}, \tag{67}$$

$$\Gamma^{(\mathrm{EP})}(B_c \to D_{s1}\gamma) = \frac{G_F^2 \alpha |C_7|^2 |V_{tb}V_{ts}^*|^2}{1024\pi^4}\left(\frac{m_{B_c}^2 - m_{D_{s1}}^2}{m_{B_c}}\right)^3 \times (16(m_b + m_s)^2 + (m_b - m_s)^2) \times [T_2(0)]^2, \tag{68}$$

$$\begin{aligned}\Gamma^{(\mathrm{total})}(B_c \to D_{s1}\gamma) = {} & \frac{G_F^2 \alpha}{1024\pi^4}\left(\frac{m_{B_c}^2 - m_{D_{s1}}^2}{m_{B_c}}\right)^3 \left\{ 64\pi^4 |V_{cb}V_{cs}^*|^2 \left[f_{B_c}^2 \{(F_A^{(D_{s1})})^2 + (F_V^{(D_{s1})})^2\} + 2 f_{B_c} f_{D_{s1}} F_V^{(B_c)} F_V^{(D_{s1})} \frac{m_{D_{s1}}}{m_{B_c}^2} \right.\right. \\ & \left. + f_{D_{s1}}^2 m_{D_{s1}}^2 \left\{ \frac{(F_A^{(B_c)})^2}{m_{B_c}^4} + \frac{(F_V^{(B_c)})^2}{m_{B_c}^4} \right\} \right] + |C_7|^2 |V_{tb}V_{ts}^*|^2 (16(m_b - m_s)^2 + (m_b + m_s)^2)[T_2(0)]^2 \\ & + 16\pi^2 T_2(0) |V_{cb}V_{cs}^*||V_{tb}V_{ts}^*| \left[f_{D_{s1}} m_{D_{s1}} X \left\{ 4 \frac{F_A^{(B_c)}}{m_{B_c}^2}(m_b - m_s) + \frac{F_V^{(Bc)}}{m_{B_c}^2}(m_b + m_s) \right\} \right. \\ & \left.\left. + f_{B_c} \{ F_V^{(D_{s1})}(m_b + m_s) X - 4 F_A^{(D_{s1})}(m_b - m_s) Y \} \right] \right\}, \end{aligned} \tag{69}$$

where X and Y are the real and imaginary parts of the Wilson coefficient C_7, respectively. In these formulas, as we previously mentioned, the fixed-point values of the form factors are used.

Finally, the numerical values of the corresponding branching ratios for the radiative decay under consideration are obtained as follows:

$$\mathbf{B}^{(\mathrm{EP})}(B_c \to D_{s1}\gamma) = (1.769 \pm 0.582) \times 10^{-8},$$
$$\mathbf{B}^{(\mathrm{WA})}(B_c \to D_{s1}\gamma) = (2.243 \pm 0.736) \times 10^{-5}, \tag{70}$$
$$\mathbf{B}^{(\mathrm{total})}(B_c \to D_{s1}\gamma) = (2.351 \pm 0.795) \times 10^{-5},$$

where the dominant contribution to each channel comes from the perturbative part. From these values, we also see that the $B_c \to D_{s1}(2460)\gamma$ transition proceeds mostly via the WA mode. The order of the total branching ratio indicates that this decay channel can be detected at LHCb in the near future. Any measurement of this decay and comparison of the obtained data with our predictions in the present work can give valuable information about the nature and internal structure of the participating particles, especially the D_{s1} meson.

ACKNOWLEDGMENTS

A. R. O. would like to thank R. Khosravi and S. Zarepour for useful discussions. Also, the partial support of the Shiraz University research council is appreciated.

APPENDIX

The explicit expressions for $C^i_{G^2}$ are given as follows:

$$\begin{aligned} C^a_{G^2} &= m_b\{-2m_b m_c^2[I_0[1,3,1] - 3m_b^2 I_0[1,4,1] + 2m_b^2(m_b - m_s)(m_b + m_s)I_0[1,5,1]] + 2m_c(I_0[1,2,1] \\ &\quad - 4m_b^2 I_0[1,3,1] - m_b m_s I_0[1,3,1] + 5m_b^4 I_0[1,4,1] + 3m_b^3 m_s I_0[1,4,1] - 3m_b^2 m_s^2 I_0[1,4,1] \\ &\quad + 2m_b^3(-m_b + m_s)(m_b + m_s)^2 I_0[1,5,1]) + 2m_s[I_0[1,2,1] - 2m_b^6 I_0[1,5,1] + m_b^4(5I_0[1,4,1] \\ &\quad + 2I_0^{[0,1]}[1,5,1]) + m_b^2(-4I_0[1,3,1] - 3I_0^{[0,1]}[1,4,1] + 3I_0^{[1,0]}[1,3,1])] + m_b(3I_0^{[0,1]}[1,3,1] \\ &\quad + I_0^{[0,2]}[1,4,1] + I_0^{[1,0]}[1,3,1] + 2m_b^4(3I_0^{[0,1]}[1,5,1] + I_0^{[1,0]}[1,5,1]) - m_b^2(9I_0^{[0,1]}[1,4,1] \\ &\quad + 2I_0^{[0,2]}[1,5,1] + 3I_0^{[1,0]}[1,4,1] - 2I_0^{[2,0]}[1,4,1]) - 4I_3^{[0,1]}[1,5,1] + 4I_3^{[1,0]}[1,3,1])\}, \end{aligned} \tag{A1}$$

$$\begin{aligned} C^b_{G^2} &= 7m_b^2 m_c^2 m_s^2 I_0[1,1,3] + m_b^3 I_0[1,1,4] + m_c I_0[1,1,4] + m_b m_c I_0[1,1,4] - m_b^2 m_c I_0[1,1,4] \\ &\quad - m_b^3 m_s I_0[1,1,4] - m_c m_s I_0[1,1,4] + 2m_b m_c m_s I_0[1,1,4] + m_b^2 m_c m_s I_0[1,1,4] \\ &\quad - m_c I_0[1,1,5] - m_b m_c I_0[1,1,5] + m_b^3 m_c I_0[1,1,5] - m_b^2 m_c^2 I_0[1,1,5] + m_c m_s I_0[1,1,5] \\ &\quad - 2m_b m_c m_s I_0[1,1,5] + 2m_b^3 m_c m_s I_0[1,1,5] - 2m_b^2 m_c^2 m_s I_0[1,1,5] - m_b I_0^{[0,1]}[1,1,4] \\ &\quad + m_s I_0^{[0,1]}[1,1,4] - 3/2 I_0^{[0,1]}[1,1,5] + 3/2 m_b^2 I_0^{[0,1]}[1,1,5] + m_b m_s I_0^{[0,1]}[1,1,5] \\ &\quad - m_s I_0^{[0,2]}[1,1,2] + 3/2 I_0^{[1,0]}[1,1,3] + m_b I_0^{[1,0]}[1,1,3] + 3m_s I_0^{[1,0]}[1,1,3] \\ &\quad + 1/2 m_b^2 I_0^{[1,0]}[1,1,4] - m_b m_s I_0^{[1,0]}[1,1,4] + 1/2 I_0^{[2,0]}[1,1,4] + m_s I_0^{[2,0]}[1,1,4], \end{aligned} \tag{A2}$$

$$\begin{aligned} C^c_{G^2} &= 1/6\{2m_b^5[m_s(-I_0[3,1,1] + m_c^2 I_0[3,1,2] + I_0^{[0,1]}[3,1,1]) + m_c(m_c^2 I_0[3,2,2] + I_0^{[0,1]}[3,1,2])] \\ &\quad - 3I_0^{[0,1]}[3,2,1] + I_0^{[0,1]}[3,2,2] + 3I_0^{[0,2]}[3,1,2] - 2I_0^{[0,2]}[3,2,1] + I_0^{[0,3]}[3,2,2] + 3I_0^{[1,0]}[3,1,1] \\ &\quad - I_0^{[1,0]}[3,1,2] + 2m_c^3 m_s\{I_0[3,2,1] - I_0[3,2,2] - 4I_0^{[0,1]}[3,1,2] + 3I_0^{[0,1]}[3,2,1] + 3I_0^{[1,0]}[3,1,1]2I_0^{[1,0]}[3,2,1]\} \\ &\quad - I_0^{[1,1]}[3,1,1] + I_0^{[1,1]}[3,1,2] - 2m_b^3[-m_c(-2I_0^{[0,1]}[3,2,2] + I_0^{[1,0]}[3,1,2] + m_c^2(I_0[3,1,2] \\ &\quad - 2I_0[3,2,2] - 3(I_0^{[0,1]}[3,2,2] + I_0^{[1,0]}[3,2,2])) + I_0^{[1,1]}[3,2,1]) + m_s((1 + 2m_c^2)I_0[3,2,2] + 2I_0^{[0,1]}[3,1,1] \\ &\quad - 2I_0^{[0,1]}[3,2,1] + 4m_c^2 I_0^{[0,1]}[3,2,2] + I_0^{[0,2]}[3,1,2] + I_0^{[2,1]}[3,1,2])) - m_b^2(2m_c^6 I_0[3,2,2] + I_0^{[0,1]}[3,1,2] \\ &\quad - 6I_0^{[0,1]}[3,2,1] - 2I_0^{[0,2]}[3,1,2] + 6I_0^{[0,2]}[3,2,2] + I_0^{[0,3]}[3,2,2] + 2m_c^3 m_s(I_0[3,1,2] - 5I_0^{[0,1]}[3,2,2] \\ &\quad - I_0^{[1,0]}[3,2,1]) + 3I_0^{[1,0]}[3,2,1] - 2I_0^{[1,0]}[3,2,2] - 7m_c^4 I_0^{[1,0]}[3,2,2], +2m_c m_s(I_0[3,1,2] - I_0[3,2,1] \\ &\quad + 2I_0^{[0,2]}[3,1,2] - I_0^{[1,1]}[3,2,1]) + 2I_0^{[1,1]}[3,2,1] - 3I_0^{[1,2]}[3,2,2] + 2I_0^{[2,0]}[3,2,1] - 2I_0^{[2,1]}[3,2,2]) \\ &\quad + I_0^{[2,1]}[3,2,2] + m_c^2(-2I_0^{[0,1]}[3,2,2] + 10I_0^{[0,2]}[3,1,2] - 2I_0^{[0,2]}[3,2,1] + I_0^{[1,0]}[3,2,1] \\ &\quad + I_0^{[1,0]}[3,2,2] - 14I_0^{[1,1]}[3,1,2] + 14I_0^{[1,1]}[3,2,2] + 3I_0^{[1,2]}[3,2,2] + 2I_0^{[2,0]}[3,1,2] \\ &\quad - 10I_0^{[2,0]}[3,2,1] - 3I_0^{[2,1]}[3,2,2] - 3I_0^{[3,0]}[3,2,2]) - I_0^{[3,0]}[3,2,2]\}, \end{aligned} \tag{A3}$$

$$\begin{aligned} C^d_{G^2} &= 1/12\{2m_c^4 I_0[3,2,1] + 2m_c^3 m_s I_0[3,1,1] + 24m_b^7(m_c + m_s)I_0[3,2,2] + I_0^{[0,1]}[3,2,2] + 2m_b^6(8m_c m_s I_0[3,1,1] \\ &\quad + 8m_c^2 I_0[3,2,2] + 3I_0^{[0,1]}[3,1,2] + I_0^{[1,0]}[3,1,1]) + 6m_b^5(2m_c^3 I_0[3,1,1] + 2m_c^2 m_s I_0[3,2,2] \\ &\quad - 2m_s(-2I_0[3,1,2] + 2m_s^2 I_0[3,2,1] + 2I_0^{[0,1]}[3,2,2]) + m_c - 9I_0^{[0,1]}[3,2,1] + I_0^{[1,0]}[3,1,2])) \\ &\quad + 3I_0^{[1,0]}[3,2,1] - I_0^{[1,0]}[3,2,2] - 2m_c m_s(2I_0^{[0,1]}[3,1,2] - I_0^{[1,0]}[3,1,1] - I_0^{[1,0]}[3,1,2]) + 2I_0^{[1,1]}[3,2,2] \\ &\quad + I_0^{[1,2]}[3,1,2] - m_c^2(-2I_0[3,1,2] + 2I_0[3,2,2] + 2I_0^{[0,1]}[3,2,2] + I_0^{[0,2]}[3,2,2] - 5I_0^{[1,0]}[3,2,1] \\ &\quad + 3I_0^{[1,0]}[3,2,2] - I_0^{[2,0]}[3,2,2]) - I_0^{[2,0]}[3,2,2] + m_b^4(4m_c^4 I_0[3,2,1] + 4m_c^3 m_s I_0[3,2,2] + 2I_0^{[0,1]}[3,2,1] \\ &\quad - 15I_0^{[0,1]}[3,2,2] - 2I_0^{[0,2]}[3,1,1] + 5I_0^{[0,1]}[3,2,2] - 4I_0^{[1,0]}[3,1,2]) + 6I_0^{[1,0]}[3,2,2] \end{aligned}$$

$$+ 3(I_0^{[0,1]}[3,2,2] + I_0^{[1,0]}[3,2,2]) + 6I_0^{[1,1]}[3,1,2] + 4I_0^{[2,0]}[3,2,2]) + 3m_b(2m_c^2 m_s(-I_0[3,2,1] - 2I_0^{[0,1]}[3,2,2] + 2I_0^{[1,0]}[3,2,2]) - 4I_0[3,2,2], +3I_0^{[0,1]}[3,2,1] - 9I_0^{[0,1]}[3,2,2] - 3I_0^{[0,2]}[3,1,2] + 5I_0^{[1,0]}[3,1,2] + I_0^{[1,0]}[3,2,1] + 3I_0^{[2,0]}[3,1,2]) - 4m_s(I_0[3,1,2] + I_0^{[0,1]}[3,2,2] + I_0^{[1,0]}[3,1,2] - 2I_0^{[1,0]}[3,2,1] - I_0^{[1,1]}[3,2,1] + I_0^{[2,0]}[3,2,2]) - 3m_b^3(-4m_c^3 I_0[3,2,2] + 2m_c^2 m_s - 4I_0^{[0,1]}[3,2,2] + 4I_0^{[1,0]}[3,1,2]) - 4m_s(-3I_0[3,2,1] + 4I_0[3,2,2] + 3I_0^{[0,1]}[3,2,2] - 4I_0^{[1,0]}[3,2,2] - 2I_0^{[1,1]}[3,2,2] + 2I_0^{[2,0]}[3,2,2]) + m_c(-16I_0[3,1,2] + 12I_0[3,2,2] - 27I_0^{[0,1]}[3,1,2] + 6I_0^{[0,1]}[3,2,1] - 6I_0^{[0,2]}[3,2,1] + 3I_0^{[1,0]}[3,1,1] + 10I_0^{[1,0]}[3,2,2] + 6I_0^{[2,0]}[3,2,2]) + m_b^2(m_c^4(-6I_0[3,1,2] + 4I_0[3,2,2]) + m_c^3(4m_s I_0[3,1,2] - 6m_s I_0[3,2,1]) - 3I_0^{[0,1]}[3,1,1] + 12I_0^{[0,1]}[3,1,2] - 9I_0^{[1,0]}[3,1,2] + 4I_0^{[1,0]}[3,2,1] + 2m_c m_s(7I_0[3,1,1] + 9I_0^{[0,1]}[3,2,2] - 2I_0^{[1,0]}[3,2,1] - 6I_0^{[1,0]}[3,2,2]) - 9I_0^{[1,1]}[3,1,1] - 2I_0^{[1,2]}[3,2,1] + m_c^2(14I_0[3,1,2] - 12I_0[3,2,2] + 9I_0^{[0,1]}[3,1,2] - 2I_0^{[0,1]}[3,2,2] + 2I_0^{[0,2]}[3,1,2] - 10I_0^{[1,0]}[3,1,2] + 9I_0^{[1,0]}[3,2,1] - 2I_0^{[2,0]}[3,1,2]) - 6I_0^{[2,0]}[3,1,2] + 6I_0^{[2,0]}[3,2,2] + 2I_0^{[3,0]}[3,1,0]) - I_0^{[3,0]}[3,2,2]\}, \tag{A4}$$

$$C_{G^2}^e = 1/6\{-2m_c m_s(-I_0[1,3,2] + I_0[1,3,3]) + 4m_b^5(-m_c I_0[1,2,3] - 2m_s I_0[1,3,3]) - 2m_c^2[I_0[1,2,2] - I_0[1,3,3]] - 3I_0^{[0,1]}[1,1,3] + I_0^{[0,1]}[1,2,2] - I_0^{[0,2]}[1,2,3] - I_0^{[1,0]}[1,2,2] + 3I_0^{[1,0]}[1,3,1] - 2m_b^4(-2m_c m_s I_0[1,2,1] - 2m_c^2 I_0[1,3,3]) + 3I_0^{[0,1]}[1,3,3] + I_0^{[1,0]}[1,2,2]) - 2m_b^3[m_c(-3I_0[1,2,3] + 2I_0[1,3,3]) + 2m_s(-I_0[1,3,3] - 2I_0^{[0,1]}[1,2,2] + 2I_0^{[1,0]}[1,3,2])] + 2m_b\{m_c[-I_0[1,2,2] + I_0[1,3,2]] + 2m_s[-I_0[1,2,3] + I_0[1,3,2] - I_0^{[0,1]}[1,1,3] + I_0^{[1,0]}[1,2,3] - 3I_0[1,3,1] + 2I_0^{[0,1]}[1,2,3] - 2I_0^{[1,0]}[1,3,3])]\} + I_0^{[2,0]}[1,3,2] + m_b^2(2m_c^2(2I_0[1,2,3] - 3I_0[1,3,2]) + 2m_c m_s(2I_0[1,1,2] - 3I_0[1,3,3]) + 9I_0^{[0,1]}[1,2,3] - 2I_0^{[0,1]}[1,3,2] + 2I_0^{[0,2]}[1,3,3] - 6I_0^{[1,0]}[1,1,3] + 3I_0^{[1,0]}[1,3,2] - 2I_0^{[2,0]}[1,3,3])\}, \tag{A5}$$

$$C_{G^2}^f = 2/3m_s\{m_b^3(-m_c^2 I_0[2,1,4]) + m_b^2 m_c(m_c^2 I_0[2,1,4] - 2I_0^{[0,1]}[2,1,4]) + m_b(I_0^{[0,1]}[2,1,3] - 2I_0^{[0,1]}[2,1,4] + I_0^{[0,2]}[2,1,2] + I_0^{[1,0]}[2,1,3] + m_c^2 I_0^{[1,0]}[2,1,3] - I_0^{[1,1]}[2,1,4]) + m_c(2I_0^{[0,1]}[2,1,3] + I_0^{[0,2]}[2,1,3] - 3I_0^{[1,0]}[2,1,4])\}, \tag{A6}$$

where we have ignored terms with higher powers of the strange quark mass. The functions $I_n[a,b,c]$ and $I_n^{[i,j]}[a,b,c]$ are defined as

$$\begin{aligned}
I_0[a,b,c] &= \frac{(-1)^{a+b+c}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b}(M_2^2)^{2-a-c}\boldsymbol{U}_0(a+b+c-4, 1-c-b),\\
I_1[a,b,c] &= \frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b}(M_2^2)^{3-a-c}\boldsymbol{U}_0(a+b+c-5, 1-c-b),\\
I_2[a,b,c] &= \frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{3-a-b}(M_2^2)^{2-a-c}\boldsymbol{U}_0(a+b+c-5, 1-c-b),\\
I_n^{[i,j]}[a,b,c] &= [M_1^2]^i[M_2^2]^j\frac{d^i}{d(M_1^2)^i}\frac{d^j}{d(M_2^2)^j}[M_1^2]^i[M_2^2]^j I_n[a,b,c].
\end{aligned} \tag{A7}$$

where $\boldsymbol{U}_0(a,b)$ is given by

$$\boldsymbol{U}_0(a,b) = \int_0^1 dy(y + M_1^2 + M_2^2)^a y^b \exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right], \tag{A8}$$

and

$$B_{-1} = \frac{m_b^2}{M_1^2}[M_1^2 + M_2^2], \qquad B_0 = \frac{1}{M_1^2 M_2^2}[M_1^2 m_c^2 + M_2^2(m_c^2 + m_b^2)], \qquad B_1 = \frac{m_c^2}{M_1^2 M_2^2}. \tag{A9}$$

[1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).

[2] P. Colangelo and A. Khodjamirian, in *At the Frontier of Particle Physics/Handbook of QCD*, edited by M. Shifman (World Scientific, Singapore, 2001), Vol. 3, p. 1495.

[3] K. Azizi and V. Bashiry, Phys. Rev. D **76**, 114007 (2007).

[4] T. M. Aliev and M. Savci, Phys. Lett. B **434**, 358 (1998).

[5] T. M. Aliev and M. Savci, Phys. Lett. B **480**, 97 (2000).

[6] K. Azizi, R. Khosravi, and V. Bashiry, Eur. Phys. J. C **56**, 357 (2008).

[7] R. Khosravi, K. Azizi, M. Ghanaatian, and F. Falahati, J. Phys. G **36**, 095003 (2009).

[8] V. V. Kiselev, arXiv:hep-ph/0211021; arXiv:hep-ph/0308214.

[9] V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, Nucl. Phys. **B585**, 353 (2000).

[10] A. Khodjamirian and D. Wyler, in *From Integrable Models to Gauge Theories*, edited by V. Gurzadian and A. Sedrakyan (World Scientific, Singapore, 2002).

[11] A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. B **358**, 129 (1995).

[12] V. A. Nestrenko and A. V. Raryushkin, Sov. J. Nucl. Phys. **39**, 811 (1984).

[13] J. Rohrwild, Phys. Rev. D **75**, 074025 (2007).

[14] P. Ball, V. M. Braun, and N. Kivel, Nucl. Phys. **B649**, 263 (2003).

[15] R. E. Cutkosky, J. Math. Phys. (N.Y.) **1**, 429 (1960).

[16] M. Q. Huang, Phys. Rev. D **69**, 114015 (2004).

[17] B. L. Ioffe, Prog. Part. Nucl. Phys. **56**, 232 (2006).

[18] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).

[19] P. Colangelo, G. Nardulli, and N. Paver, Z. Phys. C **57**, 43 (1993).

[20] V. V. Kiselev and A. V. Tkabladze, Phys. Rev. D **48**, 5208 (1993).

[21] T. M. Aliev and O. Yilmaz, Nuovo Cimento A **105**, 827 (1992).

[22] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. **B312**, 509 (1989).

[23] C. Greub, T. Hurth, M. Misiak, and D. Wyler, Phys. Lett. B **382**, 415 (1996).