

**Analysis of  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition in QCD**T. M. Aliev,<sup>1,\*,\dagger</sup> K. Azizi,<sup>2,\ddagger</sup> and M. Savcı<sup>1,\S</sup><sup>1</sup>*Physics Department, Middle East Technical University, 06531 Ankara, Turkey*<sup>2</sup>*Physics Department, Faculty of Arts and Sciences, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey*

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The  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition form factors are investigated within the light-cone QCD sum rules method. Using the most general form of the interpolating current of  $\Sigma^0$  baryon and the distribution amplitudes of  $\Lambda$  baryon we calculate the  $Q^2$  dependence of the electromagnetic form factors. Our results are compared with the predictions of the covariant spectator quark model.

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**I. INTRODUCTION**

The investigation of electromagnetic form factors of hadrons plays a key role in understanding their internal structure. The form factors measured in experiments describe the spatial distribution of charge and magnetization of hadrons [1], and indicate the deviation of hadron structure from the pointlike particle. At present, the studies are mainly focused on the nucleon form factors. Recent experimental and theoretical progress on this subject can be found in [1,2] and references therein.

The study of electromagnetic form factors of the ground state spin-1/2 baryons receives special interest. However, except the proton and neutron, the electromagnetic form factors of other members, the octet baryons, have not yet been measured. The main difficulty can be attributed to the unstable nature of the baryons containing a strange quark. From a theoretical point of view, the main problem is related to the fact that the formation of hadrons belongs to the non-perturbative region of QCD where the perturbative approach does not work. For this reason some nonperturbative approaches are needed in order to calculate these form factors, and the QCD sum rules method is recognized to be the most predictive one among all other nonperturbative approaches. Another advantage of the QCD sum rules method is that it is based on the fundamental QCD Lagrangian.

The nucleon electromagnetic form factors are calculated in framework of the light-cone version QCD sum rules method for the Ioffe and general currents in [3,4]. The electromagnetic form factors of  $\Lambda$ ,  $\Sigma$ , and  $\Xi$  baryons are studied for the Chernyak-Zhitnisky and Ioffe currents in [5]. The electromagnetic form factors of octet baryons for the most general form of the interpolating currents are studied within the light-cone QCD sum rules method in [6]. It should be noted here that the electromagnetic form factors of nucleons and other members of octet baryons have already been studied in numerous works within the

framework of lattice calculations (see [7] and references therein), and relativistic constituent quark model [8].

In the present work, we study the electromagnetic transition form factors of the  $\gamma^* \Lambda \rightarrow \Sigma^0$  in the framework of the light-cone QCD sum rules method using the most general form of the interpolating current for the  $\Sigma^0$  baryon. This decay is studied in the framework of the nonrelativistic quark model and general QCD parametrization method [9], the covariant spectator quark model [10], chiral perturbative theory [11,12], chiral quark model [13], and Skyrme model [14]. The  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition is interesting in several respects: it is unique between two different baryons that belong to the same octet family even in an exact isospin symmetry case. The second interesting peculiarity of this transition is that having different initial and final baryons is contrary to the case observed in elastic scattering of the octet baryons. For these reasons, the electric charge form factor  $G_E(Q^2)$  at  $Q^2 = 0$  should vanish. Hence, the value of  $G_E(Q^2)$  is expected to be small in its dependence on  $Q^2$ . Therefore, investigation of the  $Q^2$  dependence of the form factors receives special interest. It should be noted that the magnetic moment for the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition is investigated within the light-cone QCD sum rules method in [15]. The modern status of QCD and particularly the QCD sum rules for baryons is presented in great detail in [16].

The structure of this paper is organized as follows. In Sec. II, we derive sum rules for the form factors of the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition. In Sec. III, we present our numerical results and conclusions.

**II. SUM RULES FOR  $\gamma^* \Lambda \rightarrow \Sigma^0$  TRANSITION FORM FACTORS**

The transition form factors for  $\gamma^* \Lambda \rightarrow \Sigma^0$  are determined by the matrix element of the electromagnetic current between the  $\Lambda$  and  $\Sigma^0$  baryons. Using the conservation of the electromagnetic current, this matrix element can be determined in the following way:

$$\langle \Sigma^0(p') | j_\mu^{\text{el}} | \Lambda(p) \rangle = \bar{u}_{\Sigma^0}(p') \left\{ F_1(Q^2) \left( \gamma_\mu - \frac{\not{q} q_\mu}{Q^2} \right) - \frac{i}{m_\Lambda + m_{\Sigma^0}} \sigma_{\mu\nu} q^\nu F_2(Q^2) \right\} u_\Lambda(p), \quad (1)$$

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where  $q = p - p'$ ,  $Q^2 = -q^2$  and  $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ . Here,  $F_1(Q^2)$  and  $F_2(Q^2)$  are the Dirac and Pauli type form factors, respectively.

Experimentally, a more convenient set of the electromagnetic form factors are the Saches form factors defined as

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(m_\Lambda + m_{\Sigma^0})^2} F_2(Q^2),$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2). \quad (2)$$

In order to calculate the form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  for the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition, we consider the following correlation function:

$$\Pi_\mu(p, q) = i \int d^4x e^{iqx} \langle 0 | T \{ \eta^{\Sigma^0}(0) j_\mu^{\text{el}}(x) \} | \Lambda(p) \rangle, \quad (3)$$

where T means the time ordering,  $|\Lambda(p)\rangle$  is the  $\Lambda$  baryon state with four-momentum  $p$ ,  $\eta^{\Sigma^0}$  is the interpolating current for the  $\Sigma^0$  baryon, i.e.,

$$\eta^{\Sigma^0} = \sqrt{2} \varepsilon^{abc} \{ (u^{aT} C s^b) \gamma_5 d^c + (d^{aT} C s^b) \gamma_5 u^c + \beta (u^{aT} C \gamma_5 s^b) d^c + \beta (d^{aT} C \gamma_5 s^b) u^c \}. \quad (4)$$

Here  $C$  is the charge conjugation operator,  $\beta$  is an arbitrary parameter, and  $j_\mu^{\text{el}}$  is the electromagnetic current defined as  $j_\mu^{\text{el}}(x) = e_u \bar{u}(x) \gamma_\mu u(x) + e_d \bar{d}(x) \gamma_\mu d(x) + e_s \bar{s}(x) \gamma_\mu s(x)$ .

(5)

The correlation function can be calculated in terms of hadrons (phenomenological part) and in terms of quark and gluon degrees of freedom. Equating these two representations of the correlation function (1) we get the sum rules for the form factors of  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition.

Saturating (1) with the hadronic states with the quantum numbers of  $\Sigma^0$  baryon and separating the ground state, for the phenomenological part, we get

$$\Pi_\mu(p, q) = \frac{\langle 0 | \eta^{\Sigma^0} | \Sigma_0(p') \rangle \langle \Sigma_0(p') | j_\mu^{\text{el}} | \Lambda(p) \rangle}{m_{\Sigma^0}^2 - p'^2} + \dots, \quad (6)$$

where  $\dots$  denotes contribution of the higher states and continuum.

The matrix element  $\langle 0 | \eta^{\Sigma^0} | \Sigma_0 \rangle$  is determined as

$$\langle 0 | \eta^{\Sigma^0} | \Sigma_0 \rangle = \lambda_{\Sigma^0} u(p'),$$

where  $\lambda_{\Sigma^0}$  is the residue of the  $\Sigma_0$  baryon. Moreover, the matrix element  $\langle \Sigma_0 | j_\mu^{\text{el}} | \Lambda(p) \rangle$  is determined as is given in Eq. (1). Using these definitions, for the phenomenological part, we get

$$\Pi_\mu^{\text{ph}} = \frac{\lambda_{\Sigma^0} (\not{p}' + m_{\Sigma^0})}{m_{\Sigma^0}^2 - p'^2} \left\{ F_1(Q^2) \left( \gamma_\mu - \frac{\not{q} q_\mu}{q^2} \right) - \frac{i}{m_\Lambda + m_{\Sigma^0}} \sigma_{\mu\nu} q^\nu F_2(Q^2) \right\} u_\Lambda(p). \quad (7)$$

We see from Eq. (7) that there appears numerous structures in determining the transition form factors  $F_1(Q^2)$  and  $F_2(Q^2)$ . For this aim we choose the structures  $p_\mu$  and  $p_\mu \not{q}$ , as a result of which, for the coefficients of the selected structures, we get

$$\Pi^{(1)} = \frac{2\lambda_{\Sigma^0} F_1(Q^2)}{m_{\Sigma^0}^2 - p'^2}, \quad \Pi^{(2)} = \frac{2}{m_{\Sigma^0} + m_\Lambda} \frac{\lambda_{\Sigma^0} F_2(Q^2)}{m_{\Sigma^0}^2 - p'^2}. \quad (8)$$

As has already been noted, these form factors are described in terms of  $\Lambda$  baryon distribution amplitudes (DAs). The  $\Lambda$  baryon matrix element of three-quark operator  $\varepsilon^{abc} \langle u_\alpha^a(a_1x) d_\beta^b(a_2x) s_\gamma^c(a_3x) | \Lambda(p) \rangle$  is given in terms of  $\Lambda$  baryon DAs. The definition of this matrix element in terms of DAs and expressions of these DAs can be found in [5].

In constructing sum rules for the transition form factors  $F_1(Q^2)$  and  $F_2(Q^2)$ , we need the expression from the QCD side. This correlation function in QCD can be calculated for large negative  $p'^2$  and  $q^2 = -Q^2$  in terms of  $\Lambda$  baryon distribution amplitudes using the operator product expansion. Matching then the coefficients of the structures  $p_\mu$  and  $p_\mu \not{q}$  in the expressions of the correlation function in the phenomenological and QCD sides, we get the sum rules for the transition form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  of the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition.

In order to enhance the ground state contribution and suppress the higher state contributions, it is necessary to perform Borel transformation on the theoretical and phenomenological parts of the correlation function. After the Borel transformation, we get the final expressions for the transition form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  as

$$F_1(Q^2) = \frac{\sqrt{2}}{4} \frac{1}{2\lambda_{\Sigma^0}} e^{m_{\Sigma^0}^2/M^2} \left\{ \int_{x_0}^1 dx \left( -\frac{\rho_2(x)}{x} + \frac{\rho_4(x)}{M^2 x^2} - \frac{\rho_6(x)}{2M^4 x^3} \right) e^{-\left(\frac{Q^2 x}{M^2 x} + \frac{m_\Lambda^2 x}{M^2}\right)} + \left[ \frac{\rho_4(x_0)}{Q^2 + m_\Lambda^2 x_0^2} - \frac{1}{2x_0} \frac{\rho_6(x_0)}{(Q^2 + m_\Lambda^2 x_0^2)M^2} + \frac{1}{2} \frac{x_0^2}{(Q^2 + m_\Lambda^2 x_0^2)} \left( \frac{d}{dx_0} \frac{\rho_6(x_0)}{x_0(Q^2 + m_\Lambda^2 x_0^2)M^2} \right) \right] e^{-s_0/M^2} \right\}, \quad (9)$$

$$F_2(Q^2) = \frac{\sqrt{2}}{4} \frac{m_{\Sigma^0} + m_\Lambda}{2\lambda_{\Sigma^0}} e^{m_{\Sigma^0}^2/M^2} \left\{ \int_{x_0}^1 dx \left( -\frac{\rho'_2(x)}{x} + \frac{\rho'_4(x)}{M^2 x^2} - \frac{\rho'_6(x)}{2M^4 x^3} \right) e^{-\left(\frac{Q^2 x}{M^2 x} + \frac{m_\Lambda^2 x}{M^2}\right)} + \left[ \frac{\rho'_4(x_0)}{Q^2 + m_\Lambda^2 x_0^2} - \frac{1}{2x_0} \frac{\rho'_6(x_0)}{(Q^2 + m_\Lambda^2 x_0^2)M^2} + \frac{1}{2} \frac{x_0^2}{(Q^2 + m_\Lambda^2 x_0^2)} \left( \frac{d}{dx_0} \frac{\rho'_6(x_0)}{x_0(Q^2 + m_\Lambda^2 x_0^2)M^2} \right) \right] e^{-s_0/M^2} \right\}, \quad (10)$$

where

$$\begin{aligned}
\rho_6(x) &= 4e_u m_\Lambda^3 (1 + \beta)x(m_\Lambda^2 x^2 + Q^2)\check{\check{B}}_6(x) + 4e_d m_\Lambda^3 (1 + \beta)x(m_\Lambda^2 x^2 + Q^2)\check{\check{B}}_6(x) \\
&\quad + 8e_s m_\Lambda^2 \{m_\Lambda^2 m_s (1 - \beta)x^2 \hat{C}_6 + (1 + \beta)[m_\Lambda x(m_\Lambda^2 x^2 + Q^2)\hat{B}_6 - m_s(Q^2 \hat{B}_6 + 2m_\Lambda^2 x^2 \hat{B}_8)]\}(x), \\
\rho_4(x) &= e_u m_\Lambda \{-2m_\Lambda^2 x[2(1 - \beta)\check{C}_6 - (1 + \beta)(2\check{B}_6 - 5\check{B}_8)](x) + [2(1 - \beta)(m_\Lambda^2 x^2(\check{D}_5 - \check{C}_4 + 2\check{C}_5) - Q^2(\check{D}_2 - \check{C}_2)) \\
&\quad + (1 + \beta)(Q^2(3\check{B}_2 + 7\check{B}_4) + m_\Lambda^2 x^2(2\check{H}_1 - 2\check{E}_1 - \check{B}_2 + \check{B}_4 - 10\check{B}_5 - 20\check{B}_7))](x) \\
&\quad - 2m_\Lambda^2 x \int_0^{\bar{x}} dx_3 [2(1 - \beta)V_1^M + 5(1 + \beta)T_1^M](x, 1 - x - x_3, x_3)\} \\
&\quad + e_d m_\Lambda \{-2m_\Lambda^2 x[2(1 - \beta)\check{C}_6 - (1 + \beta)(2\check{B}_6 - 5\check{B}_8)](x) + [(1 - \beta)(-2m_\Lambda^2 x^2(\check{D}_5 + \check{C}_4 - 2\check{C}_5) \\
&\quad + Q^2(\check{D}_2 + \check{C}_2)) + (1 + \beta)(Q^2(3\check{B}_2 + 7\check{B}_4) - m_\Lambda^2 x^2(2\check{H}_1 - 2\check{E}_1 + \check{B}_2 - \check{B}_4 + 10\check{B}_5 + 20\check{B}_7))](x) \\
&\quad - 2m_\Lambda^2 x \int_0^{\bar{x}} dx_1 [2(1 - \beta)V_1^M + 5(1 + \beta)T_1^M](x_1, x, 1 - x_1 - x)\} \\
&\quad + 2e_s m_\Lambda \{2m_\Lambda(1 + \beta)[m_\Lambda x(2\hat{B}_6 - \hat{B}_8) - m_s \hat{B}_6](x) + [(1 - \beta)(2(m_\Lambda^2 x^2 \hat{C}_5 + Q^2 \hat{C}_2) \\
&\quad - m_\Lambda m_s x(2\hat{C}_2 - \hat{C}_4 - \hat{C}_5)) - (1 + \beta)(Q^2(\hat{B}_2 - 3\hat{B}_4) + m_\Lambda^2 x^2(\hat{B}_2 - \hat{B}_4 + 2\hat{B}_5 + 4\hat{B}_7) \\
&\quad - 4m_\Lambda m_s x(\hat{B}_4 - \hat{B}_5))](x) - 2m_\Lambda^2(1 + \beta)x \int_0^{\bar{x}} dx_1 T_1^M(x_1, 1 - x_1 - x, x)\}, \\
\rho_2(x) &= -2e_u m_\Lambda \{[(1 - \beta)(\check{D}_2 + \check{C}_2) - (1 + \beta)(\check{B}_2 - \check{B}_4)](x) + x \int_0^{\bar{x}} dx_3 [(1 - \beta)(A_3 + 2V_1 - 3V_3) \\
&\quad - (1 + \beta)(P_1 + S_1 - 5T_1 + 10T_3)](x, 1 - x - x_3, x_3)\} + 2e_d m_\Lambda \{[(1 - \beta)(\check{D}_2 - \check{C}_2) + (1 + \beta)(\check{B}_2 - \check{B}_4)](x) \\
&\quad + x \int_0^{\bar{x}} dx_1 [(1 - \beta)(A_3 - 2V_1 + 3V_3) - (1 + \beta)(P_1 + S_1 + 5T_1 - 10T_3)](x_1, x, 1 - x_1 - x)\} \\
&\quad + 4e_s \{m_\Lambda [(1 - \beta)\hat{C}_2 - (1 + \beta)(\hat{B}_2 - \hat{B}_4)](x) + \int_0^{\bar{x}} dx_1 \{(1 - \beta)(m_\Lambda x V_3 + m_s V_1) \\
&\quad + (1 + \beta)[2m_\Lambda x T_3 - (m_\Lambda x + 2m_s)T_1]\}(x_1, 1 - x_1 - x, x)\}, \\
\rho'_6(x) &= -4e_u m_\Lambda^2 (1 + \beta)(m_\Lambda^2 x^2 + Q^2)\check{\check{B}}_6(x) - 4e_d m_\Lambda^2 (1 + \beta)(m_\Lambda^2 x^2 + Q^2)\check{\check{B}}_6(x) \\
&\quad - 8e_s m_\Lambda^2 \{m_\Lambda m_s (1 - \beta)x \hat{C}_6 + (1 + \beta)[(m_\Lambda^2 x^2 + Q^2)\hat{B}_6 + m_\Lambda m_s x(\hat{B}_6 - 2\hat{B}_8)]\}(x), \\
\rho'_4(x) &= -e_u m_\Lambda^2 \{(1 + \beta)\check{\check{B}}_6(x) + 2x[(1 - \beta)(\check{D}_2 + \check{D}_5 - \check{C}_2 - \check{C}_4 + 2\check{C}_5) + (1 + \beta)(\check{H}_1 - \check{E}_1 - 2\check{B}_2 - 3\check{B}_4 - 5\check{B}_5 - 10\check{B}_7)](x) \\
&\quad + 2(1 - \beta) \int_0^{\bar{x}} dx_3 (A_1^M - V_1^M)(x, 1 - x - x_3, x_3)\} + e_d m_\Lambda^2 \{-(1 + \beta)\check{\check{B}}_6(x) + 2x[(1 - \beta)(\check{D}_2 + \check{D}_5 + \check{C}_2 + \check{C}_4 - 2\check{C}_5) \\
&\quad + (1 + \beta)(\check{H}_1 - \check{E}_1 + 2\check{B}_2 + 3\check{B}_4 + 5\check{B}_5 + 10\check{B}_7)](x) + 2(1 - \beta) \int_0^{\bar{x}} dx_1 (A_1^M + V_1^M)(x_1, x, 1 - x_1 - x)\} \\
&\quad - 2e_s m_\Lambda \{5m_\Lambda(1 + \beta)\hat{B}_6(x) + 2[(1 - \beta)(m_\Lambda x \hat{C}_5 - (m_\Lambda x + m_s)\hat{C}_2) - (1 + \beta)(m_\Lambda x(\hat{B}_4 + \hat{B}_5 + 2\hat{B}_7) \\
&\quad - m_s(\hat{B}_2 + \hat{B}_4))](x) + 2m_\Lambda(1 - \beta) \int_0^{\bar{x}} dx_1 V_1^M(x_1, 1 - x_1 - x, x)\}, \\
\rho'_2(x) &= -2e_u(1 - \beta) \int_0^{\bar{x}} dx_3 (A_1 - V_1)(x, 1 - x - x_3, x_3) + 2e_d(1 - \beta) \int_0^{\bar{x}} dx_1 (A_1 + V_1)(x_1, x, 1 - x_1 - x) \\
&\quad - 4e_s(1 - \beta) \int_0^{\bar{x}} dx_1 V_1(x, 1 - x - x_3, x_3), \tag{11}
\end{aligned}$$

where  $M^2$  is the Borel parameter and  $x_0$  is given as

$$x_0 = \frac{\sqrt{(Q^2 + s_0 - m_\Lambda^2)^2 + 4m_\Lambda^2 Q^2}}{2m_\Lambda^2}.$$

Here,  $s_0$  is the continuum threshold. In the expressions of  $\rho_i^{(\prime)}(x)$ , the functions  $\mathcal{F}(x_i)$  are defined as

$$\begin{aligned}
\check{\mathcal{F}}(x_1) &= \int_1^{x_1} dx'_1 \int_0^{1-x'_1} dx_3 \mathcal{F}(x'_1, 1-x'_1-x_3, x_3), \\
\check{\check{\mathcal{F}}}(x_1) &= \int_1^{x_1} dx'_1 \int_1^{x'_1} dx''_1 \int_0^{1-x''_1} dx_3 \mathcal{F}(x''_1, 1-x''_1-x_3, x_3), \\
\check{\mathcal{F}}(x_2) &= \int_1^{x_2} dx'_2 \int_0^{1-x'_2} dx_1 \mathcal{F}(x_1, x'_2, 1-x_1-x'_2), \\
\check{\check{\mathcal{F}}}(x_2) &= \int_1^{x_2} dx'_2 \int_1^{x'_2} dx''_2 \int_0^{1-x''_2} dx_1 \mathcal{F}(x_1, x''_2, 1-x_1-x''_2), \\
\hat{\mathcal{F}}(x_3) &= \int_1^{x_3} dx'_3 \int_0^{1-x'_3} dx_1 \mathcal{F}(x_1, 1-x_1-x'_3, x'_3), \\
\hat{\check{\mathcal{F}}}(x_3) &= \int_1^{x_3} dx'_3 \int_1^{x'_3} dx''_3 \int_0^{1-x''_3} dx_1 \mathcal{F}(x_1, 1-x_1-x''_3, x''_3).
\end{aligned} \tag{12}$$

We also use the following shorthand notations for the combinations of the distribution amplitudes:

$$\begin{aligned}
B_2 &= T_1 + T_2 - 2T_3, \\
B_4 &= T_1 - T_2 - 2T_7, \\
B_5 &= -T_1 + T_5 + 2T_8, \\
B_6 &= 2T_1 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, \\
B_7 &= T_7 - T_8, \\
B_8 &= -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8, \\
C_2 &= V_1 - V_2 - V_3, \\
C_4 &= -2V_1 + V_3 + V_4 + 2V_5, \\
C_5 &= V_4 - V_3, \\
C_6 &= -V_1 + V_2 + V_3 + V_4 + V_5 - V_6, \\
D_2 &= -A_1 + A_2 - A_3, \\
D_4 &= -2A_1 - A_3 - A_4 + 2A_5, \\
D_5 &= A_3 - A_4, \\
D_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6, \\
E_1 &= S_1 - S_2, \\
H_1 &= P_2 - P_1.
\end{aligned} \tag{13}$$

It follows from Eqs. (9) and (10) that in order to calculate the form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  the residue of the  $\Sigma^0$  baryon is needed. The general form of the interpolating current for  $\Sigma^0$  baryon leads to the following result for its residue [17]:

$$\begin{aligned}
\lambda_{\Sigma^0}^2 e^{-M_{\Sigma^0}^2/M^2} &= \frac{1}{256\pi^4} (5 + 2\beta + 5\beta^2) M^6 E_2(x) + \frac{m_s}{32\pi^2} M^2 E_0(x) \{ (5 + 2\beta + 5\beta^2) \langle \bar{s}s \rangle - 6(-1 + \beta^2) (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \} \\
&+ \frac{1}{24} \frac{m_0^2}{M^2} (1 - \beta) \{ 6(1 + \beta) \langle \bar{s}s \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) + (-1 + \beta) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \} + \frac{3m_s}{32\pi^2} m_0^2 \langle \bar{u}u \rangle \\
&+ \langle \bar{d}d \rangle (1 - \beta^2) \left\{ \gamma_E - \ln \left( \frac{M^2}{\Lambda^2} \right) \right\} - \frac{m_s}{192\pi^2} m_0^2 \{ 2(5 + 2\beta + 5\beta^2) \langle \bar{s}s \rangle - 3(-1 + \beta^2) (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \} \\
&- \frac{1}{6} (1 - \beta) \{ 3(1 + \beta) \langle \bar{s}s \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) + (-1 + \beta) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \},
\end{aligned} \tag{14}$$

where

$$E_n(x) = 1 - e^x \sum_{k=1}^n \frac{x^k}{k!}$$

describes the continuum subtraction and  $x = s_0/M^2$ . It should be noted that the masses and residues of nucleons and other members of the octet baryons, for Ioffe current ( $\beta = -1$ ) within the QCD sum rules approach, were firstly calculated in [18,19].

### III. NUMERICAL ANALYSIS OF THE SUM RULES FOR THE TRANSITION FORM FACTORS

In order to perform numerical analysis of the transition form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  within the light-cone QCD sum rules, we need to know the explicit expressions of the DAs for the  $\Lambda$  baryon, as well as the values of nonperturbative parameters entering into them. These input parameters for the  $\Lambda$  baryon are calculated within the two-point QCD sum rules method in [5] which are given as

$$\begin{aligned} f_\Lambda &= (6.0 \pm 0.3) \times 10^{-3} \text{ GeV}^2, \\ \lambda_1 &= (1.0 \pm 0.3) \times 10^{-2} \text{ GeV}^2, \\ |\lambda_2| &= (0.83 \pm 0.05) \times 10^{-2} \text{ GeV}^2, \\ |\lambda_3| &= (0.83 \pm 0.05) \times 10^{-2} \text{ GeV}^2. \end{aligned} \quad (15)$$

Other input parameters used in the numerical analysis are  $\langle \bar{u}u \rangle(1 \text{ GeV}) = \langle \bar{d}d \rangle(1 \text{ GeV}) = -(0.243 \pm 0.01)^3 \text{ GeV}^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$ ,  $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$  [20], and  $m_{\Sigma_0} = 1.192 \text{ GeV}$ .

Moreover, the sum rules for the transition form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  involve the continuum threshold  $s_0$ , Borel parameter  $M^2$ , and the arbitrary parameter  $\beta$  entering to the expression for the interpolating current of the  $\Sigma_0$  baryon. For the value of the continuum threshold we shall use  $s_0 = (2.8 \div 3.0) \text{ GeV}^2$ , which is obtained from the mass sum rules analysis [15]. The Borel parameter  $M^2$  is the auxiliary parameter and physical quantities such as  $F_1(Q^2)$  and  $F_2(Q^2)$  should be interdependent of it. The lower bound of the Borel mass is obtained from the condition that the higher states and continuum contributions should be less than 40% of the perturbative contribution, while the upper limit of  $M^2$  is determined by demanding that the light-cone expansion with increasing twist should be convergent. Numerical analysis shows that both conditions are satisfied when  $M^2$  lies in the region  $1.3 \text{ GeV}^2 \leq M^2 \leq 2.0 \text{ GeV}^2$ . In our calculations, we fix the lower bound of  $Q^2$  to be  $Q^2 = 1.0 \text{ GeV}^2$ , since above this value of  $Q^2$  the higher twist contributions are suppressed. In order to guarantee the higher resonance and continuum contributions to be smaller than the spectral density contribution, we consider the upper bound of  $Q^2$  as  $Q^2 \leq 8.0 \text{ GeV}^2$ . In Figs. 1 and 2, we depict the dependence of the magnetic and electric form factors  $G_M(Q^2)$  and  $G_E(Q^2)$  on  $Q^2$  at  $s_0 = 3 \text{ GeV}^2$ ,  $M^2 = 1.4 \text{ GeV}^2$ , and at several fixed values of  $\beta$ . From these figures we see that the

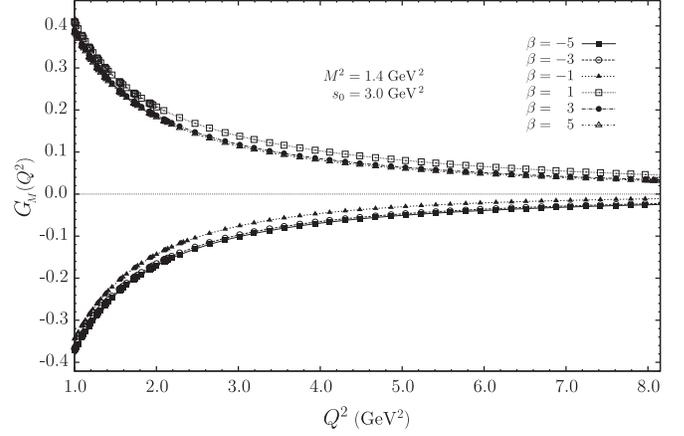


FIG. 1. The dependence of the magnetic form factor  $G_M(Q^2)$  of the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition on  $Q^2$  at  $s_0 = 3.0 \text{ GeV}^2$ ,  $M^2 = 1.4 \text{ GeV}^2$ , and at several fixed values of the arbitrary parameter  $\beta$ .

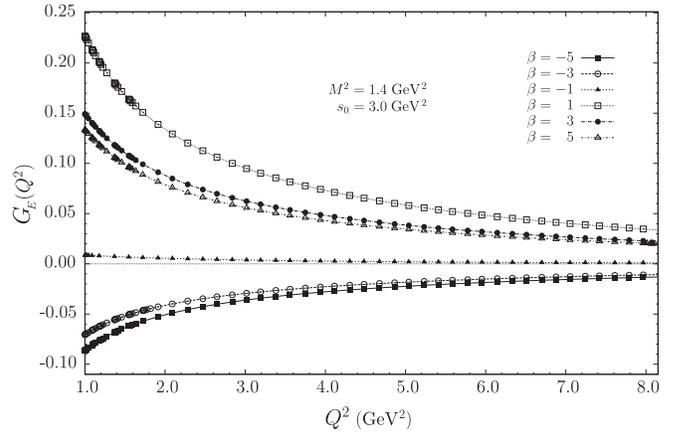


FIG. 2. The same as Fig. 1, but for the electric charge form factor  $G_E(Q^2)$ .

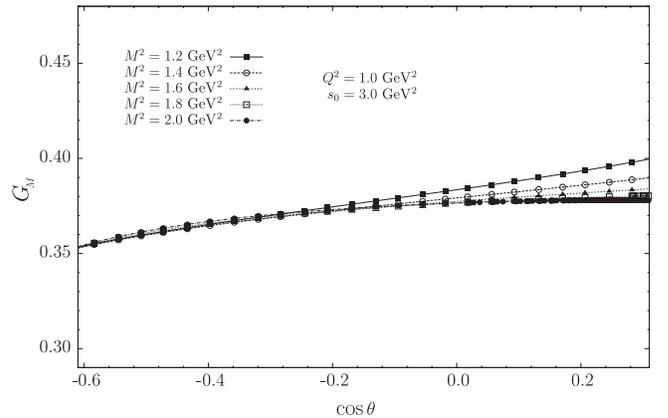


FIG. 3. The dependence of the magnetic form factor  $G_M$  of the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition on  $\cos \theta$  at  $Q^2 = 1.0 \text{ GeV}^2$ ,  $s_0 = 3.0 \text{ GeV}^2$ , and at several fixed values of the Borel mass parameter  $M^2$ .

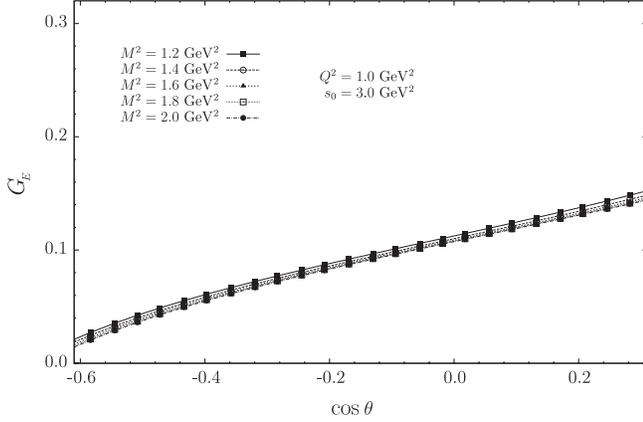


FIG. 4. The same as Fig. 2, but for the electric charge form factor  $G_E$ .

magnitude of  $G_M(Q^2)$  and  $G_E(Q^2)$  for negative (positive) values of  $\beta$  are negative (positive). Only the  $\beta = -1$  case is exceptional and at this value of  $\beta$ ,  $G_E(Q^2)$  is positive although its value is quite small and very sensitive to the values of the input parameters.

As has already been noted, the sum rules for the transition form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  contain also the auxiliary parameter  $\beta$ . For this reason we should find the “working region” of  $\beta$ , where these form factors exhibit no dependence on it. For this aim we shall work with a two-step procedure. At the first stage we use the mass sum rules for the  $\Sigma^0$  baryon analysis of which leads to the domain  $-0.6 \leq \cos \theta \leq 0.3$ , where  $\beta = \tan \theta$  (see also [17]). Having this region for  $\cos \theta$  obtained from mass sum rules, next we analyze the dependence of form factors on this parameter. Hence, we present the dependence of  $G_M(Q^2)$  and  $G_E(Q^2)$  on  $\cos \theta$  in Figs. 3 and 4 at several fixed values of other auxiliary parameters. We see from these figures that the domain  $-0.2 \leq \cos \theta \leq 0.2$  is the common region where the transition form factors are practically independent of  $\cos \theta$ .

In order to compare our predictions on the  $Q^2$  dependence of the transition form factors with the existing ones

in the literature, we note that there are only four works [7,8,10,12] in which  $Q^2$  dependence of the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition form factors are studied. In all other works, these form factors are studied only at the point  $Q^2 = 0$ . These form factors are studied up to  $Q^2 = 0.4 \text{ GeV}^2$  in [12]. Unfortunately, the light-cone sum rules method is not applicable in the region  $Q^2 < 1 \text{ GeV}^2$  and for this reason we cannot compare our results with the predictions of [12].

When we compare our results on  $G_M(Q^2)$  with those given in [8] we see that, they are very close to the prediction of [8] in the working region of  $-0.2 \leq \cos \theta \leq 0.2$ , while our results on  $G_E(Q^2)$  are larger compared to those obtained in [8]. A comparison of our results on  $G_M(Q^2)$  with the ones calculated in [10] shows that our predictions are smaller than theirs. However, the situation is contrary in the case of  $G_E(Q^2)$ , i.e., our results are larger compared to the predictions given in [10]. Therefore, checking the predictions of different approaches on the study of the  $Q^2$  dependence of the form factors for the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition receives special interest. Further improvements of our predictions on the transition form factors could be achieved by including the  $\mathcal{O}(\alpha_s)$  corrections to DAs, as well as considering possible future improvements of nonperturbative input parameters.

In conclusion, we studied the  $\gamma^* \Lambda \rightarrow \Sigma^0$  transition form factors within the light-cone QCD sum rules using the most general form of the interpolating current for the  $\Sigma^0$  baryon. We obtained the working regions for the Borel mass parameter and the arbitrary parameter  $\beta$  entering to the expressions of the interpolating current. We observed that the electric charge form factor  $G_E(Q^2)$  is quite small as expected. We also compared our results on  $G_E(Q^2)$  and  $G_M(Q^2)$  with the predictions existing in the literature. We saw that our results on  $G_M(Q^2)$  are very close to those that are calculated by the relativistic constituent quark model [8]. We further observed that our prediction on the magnetic (electric charge) form factor is smaller (larger) compared to the results of the covariant spectator quark model. The  $Q^2$  dependence of the transition form factors presented in this work can be very useful in choosing the right model.

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