Electromagnetic form factors of octet baryons in QCD

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Abstract

The electromagnetic form factors of octet baryons are estimated within light cone QCD sum rules method, using the most general form of the interpolating current for baryons. A comparison of our predictions on the magnetic dipole and electric form factors with the results of other approaches is performed.

1. Introduction

Electromagnetic form factors of nucleons provide information on their internal structures, i.e., about the spatial distribution of charge and magnetization of the nucleon. Nucleon electromagnetic form factors that are the functions of only four-momentum transfer squared \( Q^2 \) are described by Dirac \( F_1(Q^2) \) and Pauli \( F_2(Q^2) \) form factors which are related to the electric and magnetic dipole form factors \( G_E(Q^2) \) and \( G_M(Q^2) \) as

\[
G_E = F_1(Q^2) = -\frac{Q^2}{4m_N^2} F_1(Q^2),
\]

\[
G_M = F_1(Q^2) + F_2(Q^2). \tag{1}
\]

Obviously, in the limit \( Q^2 \to 0 \) the form factors \( G_E \) and \( G_M \) correspond to the charge and magnetic moments of the nucleon, while \( F_1 \) and \( F_2 \) describe the charge and anomalous magnetic moments of the nucleon.

The study of electromagnetic form factors of hadrons receives a lot of attention during the past decade. Recent experiments on the nucleon form factors using the polarized electron beam and polarized protons, which are presented in detail in [1], allow more accurate measurements of the nucleon form factors at higher values of the momentum transfer. In the polarization measurements it is observed that the ratio \( G_E^2(Q^2)/G_M^2(Q^2) \) cannot be determined by the simple dipole form \( G_D(Q^2) = (1 + Q^2/Q_0^2)^{-1} \) with \( Q_0^2 = (0.71 \text{ GeV})^2 \) [2–4]. The neutron form factors that are measured up to \( Q^2 = 3.4 \text{ GeV}^2 \) recently can provide detailed comparison of the proton and neutron form factors [5,6].

Considerable progress has also been achieved on the electromagnetic excitation of nucleon resonances during last years. The cross sections and photon asymmetries for the photo production of the pion and \( \eta \) mesons are measured at MAMI at Mainz, ELSA at Bonn, LEGS at Brookhaven, and GRAAL at Grenoble. Moreover, a large amount of data has already been collected for the \( \Delta(1232) \) excitation and single \( Q^2 \) data points are obtained for the longitudinal and transversal form factors of the \( p \to \Delta(1232), P_{11}(1440), S_{11}(1535), D_{13}(1520), \) etc., whose results are all given in [7]. These progresses in experiments open a way to real possibility of measuring the electromagnetic form factors of the octet baryons in near future.

In the present work we calculate the electromagnetic form factors of the octet baryons within the light cone QCD sum rules (LCSR) method by employing the general form of the interpolating currents. It should be noted that this problem is studied in the same method for the Ioffe current alone in [8–10]. It should also be reminded to the interested reader that the nucleon electromagnetic form factors are calculated for the Ioffe and general currents in [11] and [12], respectively.

The plan of this work is as follows. In Section 2 we introduce the correlation function which we shall use in calculating the electromagnetic form factors of the octet baryons, and discuss how the interpolating currents of the octet baryons are related to each other. In Section 3, the light cone QCD sum rules for the electromagnetic form factors are obtained in the case when the correlation functions are calculated in terms of the main nonperturbative input parameters, namely in terms of distribution amplitudes (DAs) of the octet baryons. The last section contains the details of the numerical calculations of the electromagnetic form factors of the octet baryons.

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2. LCSR for the electromagnetic form factors of octet baryons

In order to obtain the LCSR for the electromagnetic form factors of the octet baryons we start by considering the following vacuum-to-one-octet-baryon correlation function,

\[ \Pi_{\mu}(p, q) = i \int d^4xe^{iqx}\langle 0 | T \{ \eta(0) j^\mu_i(x) \} | B(p) \rangle, \]

(2)

where \( \eta \) is the interpolating current of the octet baryon, \( j^\mu_i(x) \) is the electromagnetic current, \( \mu \) is the vector Lorentz index, \( T \) is the time ordering operation, and \( B(p) \) is the one particle baryon state with momentum \( p \).

The most general forms of the interpolating currents for the octet baryons can be written as

\[ \eta^{\Sigma^0} = \sqrt{2} a_{abc} \sum_{i=1}^{2} [(u^T C A_i s^b) A_i^2 u^c + (d^T C A_i s^b) A_i^2 u^c], \]

\[ \eta^{\Sigma^+} = 2 a_{abc} (u^T C A_i s^b) A_i^2 u^c, \]

\[ \eta^{\Sigma^-} = 2 a_{abc} (d^T C A_i s^b) A_i^2 u^c, \]

\[ \eta^{\Xi^-} = \eta^{\Xi^-} (d \leftrightarrow s), \]

(3)

where \( A_1 = I, A_2 = A_2^1 = \gamma_5, A_3 = \beta \).

The interpolating current of the \( \Lambda \) baryon can also be obtained from that of \( \Sigma^0 \) baryon in the following way [13]:

\[ 2 \eta^{\Sigma^0} (d \leftrightarrow s) + \eta^{\Sigma^-} = -\sqrt{3} \eta_{\Lambda}, \]

or \[ 2 \eta^{\Sigma^0} (u \leftrightarrow s) - \eta^{\Sigma^-} = -\sqrt{3} \eta_{\Lambda}. \]

(4)

Our primary aim is the calculation of the phenomenological part of the correlation function (2). According to the standard procedure, in order to obtain the physical part of the correlation function of the octet baryons we insert a full set of baryons into Eq. (2). Separating the contribution of the ground state baryon we get

\[ \Pi_{\mu}(p, q) = \langle 0 | \eta | B(p - q) \rangle \langle B(p - q) | j^{\mu}_i | B(p) \rangle + \cdots, \]

(5)

where \( \cdots \) represents the contributions of the higher states and continuum.

The matrix element \( \langle 0 | \eta | B(p - q) \rangle \) appearing in Eq. (5) are determined as

\[ \langle 0 | \eta | B(p - q) \rangle = \lambda_B u(p - q), \]

(6)

where \( \lambda_B \) is the residue of the members of the octet baryons. The hadronic matrix element with the electromagnetic current is determined in terms of three independent form factors \( F_1, F_2 \) and \( F_3 \) in the following way:

\[ \langle B(p - q) | j^{\mu}_i | B(p) \rangle = \bar{u}(p - q) \left( F_1(q^2) \gamma^\mu_q - i \frac{\sigma^{\mu\nu} q^\nu}{2m_B} F_2(q^2) \right) u(p) + \frac{q^\mu}{2m_B} F_3(q^2) u(p). \]

(7)

From conservation of the electromagnetic current we get \( F_2(q^2) = 0 \). Taking Dirac equation into account, one can show that the general decomposition of the correlation function (2) contains six independent amplitudes in the presence of the electromagnetic current,

\[ \Pi_{\mu}(p, q) = |\Pi_1 p_{\mu} + \Pi_2 p_{\mu} q + \Pi_3 \gamma_{\mu} + \Pi_4 \gamma_{\mu} q | + |\Pi_5 q_{\mu} + \Pi_6 q_{\mu} q | u(p). \]

(8)

Using the definitions given by Eqs. (6) and (7), we get the following expression for the hadronic part,

\[ \Pi_{\mu}(p, q) = \frac{\lambda_B}{m_B^2 - (p - q)^2} \left[ 2 F_1(q^2) p_{\mu} + \frac{F_2(q^2)}{m_B} p_{\mu} q \right] + \left[ F_1(q^2) + F_2(q^2) \right] \gamma_{\mu} q \]

\[ + \left[ -2 F_1(q^2) - F_2(q^2) \right] q_{\mu} - \frac{F_2(q^2)}{2m_B} q_{\mu} q. \]

(9)

Equating the coefficients of each Lorentz structure in Eqs. (8) and (9) we get the sum rules for the form factors. In order to perform numerical analysis we need expressions of the invariant functions \( \Pi_i \) \((i = 1, \ldots, 6)\) from QCD side.

The calculation of the invariant functions \( \Pi_i \) from QCD side is carried out when the external momenta \( p \) and \( q \) are taken in deep Euclidean space, i.e., \((p - q)^2 < 0 \) and \( q^2 < 0 \), which is necessary to perform operator product expansion (OPE) near the light cone \( x^2 \sim 0 \). The OPE result can be obtained as the sum over octet baryon distribution amplitudes (DAs) of growing twist, which are the main nonperturbative inputs of the LCSR.

As has already been noted, the DAs of \( \Sigma, \Xi \) and \( \Lambda \) are investigated in [8-10]. The DAs of the octet baryons with \( J = \frac{1}{2} \) are defined from the matrix element of the three-quark operator between the vacuum and the baryon state \( | B(p) \rangle \), whose form is given as

\[ \epsilon^{abc} \langle 0 | q_1^a (a_1 x) q_2^b (a_2 x) q_3^c (a_3 x) | B(p) \rangle, \]

(10)

where \( a, b, c \) are the Dirac indices, \( a, b, c \) are the color indices, and \( a_i \) are positive numbers satisfying \( a_1 + a_2 + a_3 = 1 \). Using the Lorentz covariance, as well as spin and parity of the baryons under consideration, the matrix element (10) can be decomposed as

\[ 4 \epsilon^{abc} \langle 0 | q_1^a (a_1 x) q_2^b (a_2 x) q_3^c (a_3 x) | B(p) \rangle \]

\[ = \sum_i F_i \Gamma_{\mu}^{i \beta} (\gamma_{\mu} B(p)) \gamma_\beta, \]

(11)

where \( F_{1(2)i} \) are certain Dirac matrices, \( F_i = S_i, \gamma_i, A_i, \gamma_i \) and \( T_i \) are the DAs which do not have definite twists. The DAs with definite twists are determined from

\[ 4 \epsilon^{abc} \langle 0 | q_1^a (a_1 x) q_2^b (a_2 x) q_3^c (a_3 x) | B(p) \rangle \]

\[ = \sum_i \sqrt{i} \Gamma_{\mu}^{i \beta} (\gamma_{\mu} B(p)) \gamma_\beta, \]

(12)

where \( F_i = S_i, P_i, Ai, Vi, Ai \) and \( T_i \). The Relations among these two sets of DAs are given as

\[ S_1 = S_1, \quad (2P \cdot x) S_2 = S_1 - S_2, \]

\[ P_1 = P_1, \quad (2P \cdot x) P_2 = P_2 - P_1, \]

\[ V_1 = V_1, \quad (2P \cdot x) V_2 = V_1 - V_2 - V_3, \]

\[ V_3 = V_3, \quad (4P \cdot x) V_4 = -2V_1 + V_3 + V_4 + 2V_5, \]

\[ (4P \cdot x) V_5 = V_4 - V_3. \]

\[ (2P \cdot x)^2 V_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6. \]

\[ A_1 = A_1, \quad (2P \cdot x) A_2 = -A_1 + A_2 - A_3, \]

\[ A_2 = A_2, \quad (4P \cdot x) A_4 = -2A_1 - A_3 - A_4 + 2A_5, \]

\[ (4P \cdot x) A_5 = A_2 - A_4. \]

\[ (2P \cdot x)^2 A_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6, \]

\[ T_1 = T_1, \quad (2P \cdot x) T_2 = T_1 + T_2 - 2T_3. \]
\( 2T_3 = T_7, \quad (2P \cdot x) T_5 = T_1 - T_2 - 2T_7, \)
\( (2P \cdot x) T_5 = -T_1 + T_5 + 2T_8, \)
\( (2P \cdot x)^2 T_5 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, \)
\( (4P \cdot x) T_5 = T_7 - T_8, \)
\( (2P \cdot x)^2 T_6 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8. \)

The complete decomposition of the DAs in Eq. (11) in terms of \( S_i, T_i, A_i, V_i \) and \( T_i \) functions, as well as the explicit expressions of DAs, can all be found in [8–11].

Omitting the details of calculations of the theoretical part and equating the coefficients of the Lorentz structures \( p_\mu, q_\mu \) from hadronic and theoretical parts, and performing the Borel transformation and continuum subtraction over the variable \((p - q)^2\), we get the following sum rules for the form factors,

\[
F_1(Q^2) = \frac{L}{2 \pi \beta} \left\{ \int_0^{1-x} dx \left( \frac{\rho_2(x)}{x} - \frac{\rho_4(x)}{x^2 M^2} - \frac{\rho_6(x)}{2x^3 M^4} \right) e^{-\frac{m_0^2}{2x^2}} \right. \\
+ \left[ \frac{\rho_4(x_0)}{Q^2 + x_0^2 m_B^2} - \frac{\rho_6(x_0)}{2x_0 (Q^2 + x_0^2 m_B^2) M^2} \right] \\
+ \frac{1}{2} \left( \frac{d}{dx_0} \frac{\rho_6(x_0)}{(Q^2 + x_0^2 m_B^2) M^2} \right) \right\} e^{-\frac{(m_0^2 - m_B^2)}{2x_0^2}},
\]

where \( M \) is the Borel parameter, \( s = \frac{1}{4} Q^2 + (1 - x) m_B^2 \), \( x_0 \) is the solution of the equality \( s = m_B \), \( m_B \) is the mass of the members of the octet baryons and \( x = 1 - x \). The factor \( L \) in Eq. (13) is the normalization factor whose value for the members of the octet baryons is determined as

\[
L = \begin{cases} 
\frac{1}{7} & \text{for } \Sigma^+, \Sigma^-, \Xi^0, \Xi^-, \\
\frac{1}{2} & \text{for } \Sigma^0, \\
\frac{1}{2} \sqrt{6} & \text{for } \Lambda. 
\end{cases}
\]

The explicit expressions of \( \rho_i \) and \( \rho'_i \) for \( \Sigma^+, \Sigma^0 \) and \( \Lambda \) baryons are presented in Appendix A.

3. Numerical analysis

As has already been mentioned, the main nonperturbative inputs of LCSR are the baryon DAs. Here we would like to make the following remark about the expressions of the DAs for the \( \Sigma, \Lambda \) and \( \Xi \) baryons. In [14], the DAs for nucleons were extended up to next-to-leading order in conformal spin and the expressions of the nucleon DAs of twist-3 up to next-to-next-to leading conformal spin is found in [15]. As a result of these two works it is obtained that the nucleon form factors are sensitive to the higher conformal spin contributions. For other members of the octet baryons similar calculations are not yet done and deserves a detailed study. In present work, we consider the DAs for the \( \Lambda, \Sigma, \Xi \) baryons without these contributions, whose expressions can be found in [8–11]. The parameters appearing in the expressions of the DAs are estimated from the analysis of the sum rules given in [10–12]:

\[
f_\Sigma = (9.9 \pm 0.4) \times 10^{-3} \text{ GeV}^2, \\
\lambda_1 = -(2.1 \pm 0.1) \times 10^{-2} \text{ GeV}^2, \\
\lambda_2 = (5.2 \pm 0.2) \times 10^{-2} \text{ GeV}^2, \\
\lambda_3 = (1.7 \pm 0.1) \times 10^{-2} \text{ GeV}^2, \\
f_\Sigma = (9.4 \pm 0.4) \times 10^{-3} \text{ GeV}^2, \\
\lambda_1 = -(2.5 \pm 0.1) \times 10^{-2} \text{ GeV}^2, \\
\lambda_2 = (4.4 \pm 0.1) \times 10^{-2} \text{ GeV}^2, \\
\lambda_3 = (2.0 \pm 0.1) \times 10^{-2} \text{ GeV}^2, \\
f_\Lambda = (6.0 \pm 0.3) \times 10^{-3} \text{ GeV}^2, \\
\lambda_1 = (1.0 \pm 0.3) \times 10^{-2} \text{ GeV}^2.
\]

The remaining input parameters of the LCSR are the continuum threshold \( s_0 \), the Borel parameter \( M^2 \) and the auxiliary parameter \( \beta \) that appears in the expressions of the interpolating currents of the octet baryons.

In our numerical calculations we use the values \( s_0 = 2.5 \text{ GeV}^2, s_0 = 3.0 \text{ GeV}^2 \) and \( s_0 = 3.2 \text{ GeV}^2 \) for the continuum threshold, obtained from mass sum rules analysis in [16], for the \( \Lambda, \Sigma \) and \( \Xi \) baryons, respectively.

The Borel mass parameter \( M^2 \) is another auxiliary parameter of the sum rules. Therefore the “working region” of \( M^2 \) should be found, where the form factors are practically independent of it. The lower limit of \( M^2 \) can be obtained by requiring that the higher states and continuum contributions to the sum rules constitute, maximally, about 40% of the total result. The upper limit of \( M^2 \) can be determined by demanding that the operator product expansion should be convergent. Our calculations show that the region in which the aforementioned two conditions are properly satisfied, are: 1.2 GeV^2 \( \leq M^2 \leq 3.0 \text{ GeV}^2 \) for the \( \Lambda, \Sigma \) and \( \Xi \) baryons; and 1.4 GeV^2 \( \leq M^2 \leq 3.5 \text{ GeV}^2 \) for \( \Sigma, \Xi \) baryons. In further numerical analysis, we use \( M^2 = 2.0 \text{ GeV}^2 \) for the \( \Sigma, \Xi \) and \( \Lambda \) baryons.

The residues of the octet baryons are calculated in [16] and we shall use these results in our numerical analysis. Furthermore, it should be noted that, from experimental point of view, it is more convenient to study the Sachs form factors \( G_E \) and \( G_M \) that are given in Eq. (1).

The \( Q^2 \) dependence of the magnetic and electric form factors for \( \Sigma^+, \Sigma^- \) and \( \Lambda \) baryons are shown in Figs. 1–6. In order to get “good” convergence of the light cone expansion and reliable results from the LCSR, sufficiently large \( Q^2 \) is needed. In our numerical calculations we consider the lower limit of \( Q^2 \) as \( Q^2 = 1 \text{ GeV}^2 \), where above this point the higher twist contributions are suppressed. On the other side, the higher resonance and continuum contributions become small enough when \( Q^2 \leq 8 \text{ GeV}^2 \). For this reason, we perform numerical analysis in the region \( 1 \text{ GeV}^2 \leq Q^2 \leq 8 \text{ GeV}^2 \).

In odd-numbered Figs. 1, 3, 5 (even-numbered Figs. 2, 4, 6) we present the dependence of the magnetic dipole form factors \( G_M(Q^2) \) (electric form factor \( G_E(Q^2) \)) on \( Q^2 \), at fixed values of \( s_0 \) and \( M^2 \) chosen from their working regions, and at several fixed values of the arbitrary parameter \( \beta \), for \( \Sigma^+, \Sigma^- \) and \( \Lambda \) baryons, respectively.
Fig. 1. The dependence of the magnetic form factor $G_M(Q^2)$ of the $\Sigma^+$ baryon on $Q^2$ at $s_0 = 3.0$ GeV$^2$ and $M^2 = 2.0$ GeV$^2$, at several different fixed values of the arbitrary parameter $\beta$.

Fig. 2. The same as in Fig. 1, but for the electric charge form factor $G_E(Q^2)$.

Fig. 3. The same as in Fig. 1, but for the $\Xi^-$ baryon at $s_0 = 3.2$ GeV$^2$. 
Fig. 4. The same as in Fig. 2, but for the $\Xi^-$ baryon at $s_0 = 3.2$ GeV$^2$.

Fig. 5. The same as in Fig. 1, but for the $\Lambda$ baryon at $s_0 = 2.6$ GeV$^2$ and $M^2 = 1.8$ GeV$^2$.

Fig. 6. The same as in Fig. 5, but for the electric charge form factor $G_E(Q^2)$. 
Fig. 7. The dependence of the magnetic form factor $G_M$ of the $\Sigma^+$ baryon on $\cos \theta$ at $Q^2 = 1.0 \text{ GeV}^2$ and $s_0 = 3.0 \text{ GeV}^2$, at several different fixed values of the Borel mass parameter $M^2$.

Fig. 8. The same as in Fig. 7, but for the electric charge form factor $G_E$.

- In the case of $\Sigma^+$, these form factors get positive (negative) values for negative (positive) values of the parameter $\beta$.
- The situation for $\Sigma^-$ is contrary to the behaviors of the form factors of the $\Sigma^+$, i.e., the values of $G_M$ and $G_E$ are positive (negative) when the parameter $\beta$ is positive (negative).
- In the case of $\Sigma^0$ baryon, the form factors exhibit similar behaviors as the form factors of $\Sigma^+$ baryon.
- It is observed that the form factor $G_E$ of the $\Sigma^0$ baryon changes its sign practically at all considered values of $\beta$. The zero values of $G_E$ depend on the values of the arbitrary parameter $\beta$. But the values of $G_E$ are quite small, whose maximum value is about $G_E(Q^2) = 0.06$.
- It is interesting to observe that at $\beta = 3$ and $\beta = 5$, $G_M$ for the $\Sigma^0$ changes its sign, while for the other values of $\beta$ it is always negative.
- For positive (negative) values of $\beta$ the magnetic dipole form factor $G_M(Q^2)$ for $\Lambda$ baryon attains at positive (negative) values.
- The situation for electric form factor $G_E(Q^2)$, however, is slightly different. Namely, in the case of Ioffe current for which $\beta = -1$, $G_E(Q^2)$ becomes negative whose magnitude is negligibly small.
- When the form factors of the $\Sigma^-$ baryon are considered we see that $G_M$ changes its sign only at $\beta = 5$, while for all other values of $\beta$ it gets at only negative values. On the other hand the form factor $G_E$ is positive (negative) for all positive and negative values of $\beta$.

We now compare our results on $Q^2$ dependence of the magnetic and charge form factors with the ones existing in the literature. These form factors are discussed within the LCSR method for the Ioffe current ($\beta = -1$) in [9,10], within the framework of the fully relativistic constituent quark model in [17], in the framework of the covariant spectator quark model [18] and lattice QCD [19]. When we compare our predictions on the form factors with the results of the above-mentioned works we obtain that, at $\beta = -1$, our predictions on $G_E$ are very close to the results of [9,10], and [17], except for the $\Sigma^0$ baryon. Our predictions for the magnetic form factor $G_M$ agree within the errors with the existing results. The small differences among the predictions can be attributed to the different values of the input parameters used in the numerical analysis.

As has already been noted, the interpolating currents of the octet baryons contain also the auxiliary arbitrary parameter $\beta$. 
Obviously, the physically measurable quantities should be independent of this parameter. In order to find the working region of the parameter \( \beta \) we demand that the form factors are practically independent of it. As an example, in Figs. 7 and 8 we present the dependence of \( G_M(Q^2) \) and \( G_E(Q^2) \) on \( \cos \theta (\beta = \tan \theta) \) for \( \Sigma^+ \) baryon at fixed values of \( s_0 = 3.0 \text{ GeV}^2 \) and \( M^2 = 2.0 \text{ GeV}^2 \), for two fixed choices of \( Q^2 \), namely, \( Q^2 = 2.0 \text{ GeV}^2 \) and \( Q^2 = 3.0 \text{ GeV}^2 \). We see from these figures that, in the region \(-0.3 \leq \cos \theta \leq 0.3, G_E \) and \( G_M \) show very weak dependence on \( \beta \). In other words, the working region of the parameter \( \beta \) for the \( \Sigma^+ \) baryon is \(-0.3 \leq \cos \theta \leq 0.3 \).

We perform similar analysis for all other members of the octet baryons and find out that the region \(-0.2 \leq \cos \theta \leq 0.2 \) is the common working region to them as well. It should be noted here that \( G_M(Q^2) \) for \( \Sigma^0 \) and \( \Sigma^- \) baryons exhibit stability in the range \(-0.2 \leq \cos \theta \leq 0.2 \). Also note that \( \beta = -1 \) point, which is the loffe current corresponding to \( \cos \theta = -0.71 \), belongs to the region where the predictions for the form factors are not reliable. Choosing the values of \( M^2 \) and \( \beta \) from the relevant working regions, and from a comparison of our predictions on the form factors with the results of the above-mentioned works we see that

- Predictions of all works for \( G_M(Q^2) \) are very close to each other within the error limits;
- Our predictions on \( G_E(Q^2) \) agree with the results of other approaches, except for the \( \Sigma^0 \) and \( \Lambda \) baryons. In these cases our results are very close to the predictions of the lattice QCD, while considerable disagreements are observed with those obtained in [17] and [18].

The results obtained in this work can be improved by taking into account the \( O(a_s) \) corrections to the distribution amplitudes, and more accurate values of the input parameters entering the sum rules.

In conclusion, in the present work we have studied the charge \( G_E(Q^2) \) and magnetic dipole \( G_M(Q^2) \) form factors within the LCSR method by using the most general for of the interpolating currents for the octet baryons. We have compared our predictions on these form factors with the results existing in literature that were obtained in framework of the relativistic quark model, lattice QCD and LCSR for the loffe current.

**Appendix A**

In this appendix we present the explicit expressions of the functions \( \rho_2(\rho'_2), \rho_4(\rho'_4) \) and \( \rho_6(\rho'_6) \) entering to the sum rules for the form factors \( F_1(Q^2) \) and \( F_2(Q^2) \).

\[
\rho_6^{\Sigma^+}(x) = 4e_m m_{\Sigma^+}^2 (1 + \beta)x(m_{\Sigma^+}^2 + x^2 + Q^2) \tilde{B}_6(x)
+ 4e_m m_{\Sigma^+}^2 [m_{\Sigma^+}^2 m_5 (1 - \beta)x^2 \tilde{e}_6 + (1 + \beta)
\times [m_{\Sigma^+} (m_{\Sigma^+}^2 + x^2 + Q^2) \tilde{B}_6
- m_5 (Q^2 \tilde{B}_6 + 2m_{\Sigma^+}^2 x^2 \tilde{B}_6)])(x),
\]

\[
\rho_4^{\Sigma^+}(x) = e_m m_{\Sigma^+} \left[-2m_{\Sigma^+}^2 x[(1 - \beta)(\tilde{C}_6 + \tilde{D}_6)
- (1 + \beta)(2\tilde{B}_6 - 3\tilde{B}_7)](x)
+ (1 - \beta)[m_{\Sigma^+}^2 x^2(\tilde{D}_4 - 3\tilde{D}_5 - \tilde{C}_4 + 3\tilde{C}_5)
+ 2Q^2(\tilde{D}_2 + \tilde{C}_2)](x)
+ (1 + \beta)[Q^2(\tilde{B}_2 + 5\tilde{B}_4) - m_{\Sigma^+}^2 x^2(2\tilde{H}_1 - 2\tilde{E}_1
+ \tilde{B}_2 - \tilde{B}_4 + 6\tilde{B}_5 + 12\tilde{B}_7)](x)
\times (x, 1 - x - x_3, x_3) + e_d m_{\Sigma^+}
\times \left[-2m_{\Sigma^+} x[(1 - \beta)(\tilde{C}_6 - \tilde{D}_6) + 2(1 + \beta)\tilde{B}_8](x)
+ (1 - \beta)[-m_{\Sigma^+}^2 x^2(\tilde{D}_4 - \tilde{D}_5 + \tilde{C}_4 - \tilde{C}_5)](x)
+ (1 + \beta)[2Q^2(\tilde{B}_2 + \tilde{B}_4) - 4m_{\Sigma^+}^2 x^2(\tilde{B}_5 + 2\tilde{B}_7)](x)
\times (x_1, x, 1 - x_1 - x, x_1) + e_d m_{\Sigma^+}
\times \left[2m_{\Sigma^+} x[(1 - \beta)(\tilde{B}_6 - \tilde{B}_8) - m_1 \tilde{B}_6](x)
+ (1 - \beta)[2(m_{\Sigma^+}^2 x^2 \tilde{C}_5 + Q^2 \tilde{C}_2) - m_{\Sigma^+} m_1 x(2\tilde{C}_2 - \tilde{C}_4 - \tilde{C}_3)](x)
- (1 + \beta)[Q^2(\tilde{B}_2 - 3\tilde{B}_4 + m_{\Sigma^+}^2 x^2)
\times (\tilde{B}_2 - \tilde{B}_4 + 4\tilde{B}_5 + 4\tilde{B}_7) - 4m_{\Sigma^+} m_1 x(\tilde{B}_4 - \tilde{B}_5)](x)
\times (x_1, x, 1 - x_1 - x, x_1) + e_d m_{\Sigma^+}
\times \left[2m_{\Sigma^+} x[1 + \beta] \int_0^\infty dx_1 T^M_1(x_1, 1 - x_1 - x, x_1),
\right.
\]

\[
\rho_2^{\Sigma^+}(x) = 2e_m m_{\Sigma^+} x \int_0^\infty dx_1 [(1 - \beta)(A_1 + 2A_3 - V_1 + 2V_3)
- (1 + \beta)(P_1 + S_1 + 3T_1 - 6T_3)](x_1 - x - x_3, x_3)
+ 2e_m m_{\Sigma^+} \left[(1 - \beta)(\tilde{D}_2 - \tilde{C}_2)
- x \int_0^\infty dx_1 [(1 - \beta)(A_1 + A_3 + V_1 - V_3)
+ 2(1 + \beta)(T_1 - 2T_3)](x_1, x, 1 - x_1 - x)
+ 2e_m \left[(1 - \beta)(\tilde{C}_2 + (1 + \beta)(\tilde{B}_2 - \tilde{B}_4)](x)
\times (x_1, x, 1 - x_1 - x, x_1),
\right.
\]

\[
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\[ \rho_{6}^{(\Sigma^{0})}(x) = -4e_d m_{\Sigma^{0}}^2 (1 + \beta) (m_{\Sigma^{0}}^2 x^2 + Q^2) \hat{B}_6(x) \]
\[ - 4e_d m_{\Sigma^{0}}^2 (1 + \beta) (m_{\Sigma^{0}}^2 x^2 + Q^2) \hat{B}_6(x) \]
\[ - 8e_d m_{\Sigma^{0}}^2 (m_{\Sigma^{0}} m_0 (1 - \beta) x \hat{c}_0 + (1 + \beta) x [ (m_{\Sigma^{0}}^2 x^2 + Q^2) \hat{B}_6 + m_{\Sigma^{0}} m_0 (x \hat{B}_6 - 2 \hat{B}_8) ]) (x) \]
\[ \times (1 + \beta) (Q^2 (3 \hat{B}_2 + 7 \hat{B}_4) + m_{\Sigma^{0}}^2 x^2 (2 \hat{H}_1 - \hat{E}_1 - \hat{B}_2 + \hat{B}_4 - 10 \hat{B}_5 - 20 \hat{B}_7)) (x) \]
\[ + 2m_{\Sigma}^2 x \int_{0}^{\hat{x}} dx_3 (2 (1 - \beta) V_1^M + 5 (1 + \beta) T_1^M) \]
\[ \times (x, 1 - x - x_3, x_3) + e_d m_A \]
\[ \times \left( \begin{array}{c}
(1 - \beta) (\hat{H}_1 - \hat{E}_1 - 2 \hat{B}_2 + 3 \hat{B}_4 + 5 \hat{B}_5 + 10 \hat{B}_7) (x) \\
+ 2 \int_{0}^{\hat{x}} dx_3 (A_1^M - V_1^M) (x_1, 1 - x - x_3, x_3) \end{array} \right) \]
\[ - 2e_d m_{\Sigma^{0}}^2 (1 + \beta) \hat{B}_6(x) \]
\[ - 2e_d m_{\Sigma^{0}}^2 \int_{0}^{\hat{x}} dx_3 (A_1^M - V_1^M) (x_1, 1 - x - x_3, x_3) \]
\[ + 2 \int_{0}^{\hat{x}} dx_3 (A_1^M - V_1^M) (x_1, 1 - x - x_3, x_3) \]
\[ + 2 m_{\Sigma^{0}}^2 (1 - \beta) \int_{0}^{\hat{x}} dx_1 V_1^M (x_1, 1 - x - x, x) \]
\[ \rho_{6}^{(\Sigma^{0})}(x) = -2e_d (1 - \beta) \int_{0}^{\hat{x}} dx_3 (A_1 - V_1) (x_1, 1 - x - x_3, x_3) \]
\[ + 2e_d (1 - \beta) \int_{0}^{\hat{x}} dx_3 (A_1 + V_1) (x_1, 1 - x - x_3, x_3) \]
\[ - 4e_d (1 - \beta) \int_{0}^{\hat{x}} dx_1 V_1 (x, 1 - x - x_3, x_3) \]
\[ \rho_{6}^{(\Sigma^{0})}(x) = -12e_d m_{A}^2 (1 + \beta) x (m_{A}^2 x^2 + Q^2) \hat{B}_6(x) \]
\[ - 20 e_d m_{A}^2 (1 + \beta) x (m_{A}^2 x^2 + Q^2) \hat{B}_6(x) \]
\[ + 8 e_d m_{A}^2 (2 m_{A}^2 Q^2 \hat{B}_6 - m_{A}^2 m_0 x^2 (\hat{c}_0 - 2 \hat{B}_8)) \]
\[ - m_A (1 + \beta) x [ (m_{A}^2 x^2 + Q^2) \hat{B}_6 + m_A m_0 \hat{c}_0 ] (x) \]
\[ \rho_{6}^{(\Sigma^{0})}(x) = 3e_d m_{A} \int_{0}^{\hat{x}} dx_1 V_1 (1 - \beta) \hat{c}_0 - (1 + \beta) (2 \hat{B}_6 - 5 \hat{B}_8) \]
\[ - 3 [2 (1 - \beta) (m_{A}^2 x^2 (\hat{D}_5 - \hat{C}_4 + 2 \hat{C}_5) + Q^2 (\hat{D}_2 - \hat{C}_3)) \]
\[ + (1 + \beta) (Q^2 (3 \hat{B}_2 + 7 \hat{B}_4) + m_{A}^2 x^2 (2 \hat{H}_1 - \hat{E}_1 - \hat{B}_2 + \hat{B}_4 - 10 \hat{B}_5 - 20 \hat{B}_7)) (x) \]
\[ + 2m_{A}^2 x \int_{0}^{\hat{x}} dx_3 (2 (1 - \beta) V_1^M + 5 (1 + \beta) T_1^M) \]
\[ \times (x, 1 - x - x_3, x_3) + e_d m_A \]
\[ \times \left( \begin{array}{c}
(1 - \beta) (\hat{H}_1 - \hat{E}_1 - 2 \hat{B}_2 + 3 \hat{B}_4 + 5 \hat{B}_5 + 10 \hat{B}_7) (x) \\
+ 2 \int_{0}^{\hat{x}} dx_3 (A_1^M - V_1^M) (x_1, 1 - x - x_3, x_3) \end{array} \right) \]
\[ - 2e_d m_{A}^2 \int_{0}^{\hat{x}} dx_3 (2 (1 - \beta) \hat{c}_0 - (1 + \beta) (\hat{B}_2 - 5 \hat{B}_8) \]
\[ - m_A (1 + \beta) x (5 \hat{B}_5 + 9 \hat{B}_8) \]
\[ + [2 (1 - \beta) (m_{A}^2 x^2 (\hat{D}_5 + \hat{C}_4 - 6 \hat{C}_5) - Q^2 (\hat{D}_2 + 5 \hat{C}_2)) \]
\[ + (1 + \beta) (Q^2 (\hat{B}_2 - 19 \hat{B}_4) + m_{A}^2 x^2 (2 \hat{H}_1 - 2 \hat{E}_1 + \hat{D}_2 + 5 \hat{B}_4 + 18 \hat{B}_5 + 36 \hat{B}_7)) (x) \]
\[ + 2 m_{A}^2 x \int_{0}^{\hat{x}} dx_3 (2 (1 - \beta) V_1^M + 9 (1 + \beta) T_1^M) \]
\[ \times (x, 1 - x - x_3, x_3) \}
\[ \rho_{2}^{(\Sigma^{0})}(x) = 6e_d m_A \left\{ \begin{array}{c}
(1 - \beta) (\hat{D}_2 + \hat{C}_2) - (1 + \beta) (\hat{B}_2 - \hat{B}_4) \}
\times (x, 1 - x - x_3, x_3) \}
\[ + x \int_{0}^{\hat{x}} dx_3 (1 - \beta) (A_3 + 2 V_1 - 3 V_3) \]
\[ - \left( \begin{array}{c}
(1 + \beta) (3 \hat{C}_2) \\
- 3 (1 + \beta) (\hat{B}_2 - \hat{B}_4) \end{array} \right) \}
\[ + x \int_{0}^{\hat{x}} dx_3 (1 - \beta) (A_3 - 2 V_1 + 7 V_3) \]
\[ - \left( \begin{array}{c}
(1 + \beta) (P_1 + S_1 - 9 T_1 - 18 T_3) \}
\times (x, 1 - x - x_3, x_3) \}
\[ - 2e_d m_A \left\{ \begin{array}{c}
(1 - \beta) (\hat{D}_2 + 3 \hat{C}_2) \\
- 3 (1 + \beta) (\hat{B}_2 - \hat{B}_4) \end{array} \right) \}
\[ + x \int_{0}^{\hat{x}} dx_3 (1 - \beta) (A_3 - 2 V_1 + 7 V_3) \]
\[ - \left( \begin{array}{c}
(1 + \beta) (P_1 + S_1 + 9 T_1 - 18 T_3) \}
\times (x, 1 - x - x_3, x_3) \}
\[ - 2e_d m_A \left\{ \begin{array}{c}
(1 - \beta) (\hat{D}_2 + 3 \hat{C}_2) \\
- 3 (1 + \beta) (\hat{B}_2 - \hat{B}_4) \end{array} \right) \}
\[ + x \int_{0}^{\hat{x}} dx_3 (1 - \beta) (A_3 - 2 V_1 + 7 V_3) \]
\[ - \left( \begin{array}{c}
(1 + \beta) (P_1 + S_1 + 9 T_1 - 18 T_3) \}
\times (x, 1 - x - x_3, x_3) \}
the expressions for $\rho_i$ and $\rho_i'$, the functions $\mathcal{F}(x_i)$ are defined in the following way:

$$
\mathcal{F}(x_1) = \int_{x_1}^{x_1'} dx_1' \int_{x_1'}^{x_1''} dx_3 \mathcal{F}(x_1', 1 - x_1'' - x_3, x_3),
$$

$$
\mathcal{F}(x_2) = \int_{x_2}^{x_2'} dx_2' \int_{x_2'}^{x_2''} dx_3 \mathcal{F}(x_2', 1 - x_2'' - x_3, x_3),
$$

$$
\mathcal{F}(x_3) = \int_{x_3}^{x_3'} dx_3' \int_{x_3'}^{x_3''} dx_1 \mathcal{F}(x_1, 1 - x_1'' - x_3', x_3'),
$$

$$
\mathcal{F}(x_4) = \int_{x_4}^{x_4'} dx_4' \int_{x_4'}^{x_4''} dx_1 \mathcal{F}(x_1, 1 - x_1'' - x_3', x_3'),
$$

and the definitions of the functions $B_i$, $C_i$, $D_i$, $E_i$ and $H_1$ are given as

$$
B_2 = T_1 + T_2 - 2T_3, 
$$

$$
B_4 = T_1 - T_2 - 2T_7, 
$$

$$
B_5 = -T_1 + T_5 + 2T_8, 
$$

$$
B_6 = 2T_1 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, 
$$

$$
B_7 = T_2 - T_8, 
$$

$$
B_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8, 
$$

$$
C_2 = V_1 - V_2 - V_3, 
$$

$$
C_4 = -2V_1 + V_3 + V_4 + 2V_5, 
$$

$$
C_5 = V_4 - V_3, 
$$

$$
C_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6, 
$$

$$
D_2 = -A_1 + A_2 - A_3, 
$$

$$
D_4 = 3A_2 - A_3 - A_4 + 2A_5, 
$$

$$
D_5 = A_3 - A_4, 
$$

$$
D_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6, 
$$

$$
E_1 = S_1 - S_2, 
$$

$$
H_1 = P_2 - P_1. 
$$

The expressions of the functions $\rho_i(x)$ and $\rho_i'(x)$ for the $\Xi^0$ ($\Xi^-$) baryons can be obtained from the corresponding results of $\Sigma^+$ ($\Sigma^-$) by making the replacement $s \leftrightarrow u$ ($s \leftrightarrow d$).

References