Magnetic moments of $\Xi'_Q-\Xi_Q$ transitions in light cone QCD

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The $\Xi'_Q-\Xi_Q$ transition magnetic moments are calculated in a framework of the light cone QCD sum rules method. The values of the transition magnetic moments obtained are compared with the predictions of the other theoretical approaches.


I. INTRODUCTION

During the last decade the heavy hadron spectroscopy has been a field of permanent and growing interest. This interest has been dictated by the exciting experimental results obtained in regard to this subject. At present all ground state baryons containing a single charm quark have already been observed, and their masses are also measured. Many of the spin-1/2 hadrons such as $\Lambda_b$, $\Sigma_b$, $\Xi_b$, and $\Omega_b$, and spin-3/2 hadrons such as $\Xi_b$ have also been observed [1–3]. Recently, the spin-3/2 $\Xi_b$ baryon has been discovered [4], and the latest measurement of the lifetime of $\Lambda_b$ baryon has been announced [5]. On the other hand, the experimental situation with two and three heavy quarks has not yet been reached. Only the SELEX Collaboration announced the observation of the spin-1/2 $\Xi_{cc}$ baryon [6], which has not yet been confirmed by any other collaborations.

Experimental observation and investigation of the properties of doubly and triply heavy baryons constitute one of the most promising research areas in particle physics. One of the most static quantities that could give valuable information about the internal structure of baryons is their magnetic moments. Magnetic moments of heavy baryons have been investigated extensively within the framework of different approaches, such as the naive quark model, chiral quark model, nonrelativistic quark model, and QCD sum rules method.

The main advantage of the QCD sum rules method compared to the other nonperturbative approaches is that it is based on the fundamental QCD Lagrangian, and it takes into account the nonperturbative nature of the QCD vacuum. In order to solve the problems that are inherent in the traditional QCD sum rules method, the light cone version of the QCD sum rules is proposed (for more about this method, see for example [7]). In this version of the QCD sum rules, the operator product expansion is performed over twists of the operators, and nonperturbative effects are all encoded into the matrix elements of the nonlocal operators between the vacuum and one-particle states. The light cone QCD sum rules (LCSR) has so far been applied to many problems in hadron physics (for the very recent applications of the LCSR, see for example [8–10]).

In the present work we calculate the $\Xi'_Q-\Xi_Q$ transition magnetic moment within the QCD sum rules method. Note that the $\Lambda_Q-\Xi_Q$ transition magnetic moment was studied within the same framework in [11].

This paper is organized as follows. In Sec. II the sum rules for the $\Xi'_Q-\Xi_Q$ transition magnetic moment are obtained. The following section contains numerical calculations, discussion and comparison with the predictions of the other theoretical methods existing in literature.

II. DERIVATION OF THE SUM RULES FOR THE $\Xi'_Q-\Xi_Q$ TRANSITION MAGNETIC MOMENTS IN LCSR

We start this section by summarizing the useful information on the $SU(3)$ classification of the heavy hadrons with single heavy quarks. These baryons belong to either $SU(3)$ antisymmetric $\bar{3}_F$ or symmetric $6_F$ flavor representations. It is well known that in $6_F$ representation the ground state baryons must have total spin 1, while their total spin is 0 in $\bar{3}_F$ representation. Therefore, baryons in $6_F$ representation can have spin-1/2 or -3/2, but hadrons in $\bar{3}_F$ representation can have only spin-1/2. In the present work we consider only spin-1/2 baryons from both representations; i.e., we consider $\Xi_Q$ from $6_F$ and $\Xi_Q$ from $\bar{3}_F$ representations, respectively.

Following this brief information, we can proceed now to calculate the $\Xi'_Q-\Xi_Q$ transition magnetic moment within LCSR. For this aim we consider the following correlation function,

$$\Pi = i \int d^4e^{i p \cdot x} \langle 0 | T \{ \bar{n}_{\Xi_Q} (x) \bar{n}_{\Xi'_Q} (0) \} | 0 \rangle \gamma^\mu,$$  \hspace{1cm} (1)

where $\gamma$ is the external electromagnetic field; $\bar{n}_{\Xi_Q}$ and $\bar{n}_{\Xi'_Q}$ are the interpolating currents of $\Xi_Q$ and $\Xi'_Q$ baryons. The
general forms of the interpolating currents of $\Xi_Q$ and $\Xi'_Q$ baryons are as follows,

$$\eta_{\Xi_Q} = \frac{1}{\sqrt{6}} e^{abc} \{ (s^a T C q^b) y_5 Q^c + (s^a T C q^b) y_5 q^c \}
- (q^a T C q^b) y_5 s^c + 2t (s^a T C y_5 q^b) Q^c
+ t (s^a T C y_5 Q^b) q^c - (t (s^a T C y_5 Q^b) s^c) \},$$

$$\eta_{\Xi'_Q} = \frac{1}{\sqrt{2}} e^{abc} \{ (s^a T C q^b) y_5 q^c + (q^a T C q^b) y_5 s^c \}
+ t (s^a T C y_5 Q^b) q^c + t (q^a T C y_5 Q^b) s^c \},$$

where $a, b, c$ are the color indices; $q = u$ or $d$, and $Q = b$ or $c$ quark; $C$ is the charge conjugation operator; and $t$ is the arbitrary parameter.

It should be noted here that $\Xi_Q$ and $\Xi'_Q$ baryons do have the same quantum numbers and in principle there can be mixing between them. This mixing can be implemented by modifying the interpolating currents for physical states as follows:

$$\eta_{\Xi_Q}^{\text{phys}} = \cos \theta_Q \eta_{\Xi_Q} + \sin \theta_Q \eta_{\Xi'_Q},$$

$$\eta_{\Xi'_Q}^{\text{phys}} = - \sin \theta_Q \eta_{\Xi_Q} + \cos \theta_Q \eta_{\Xi'_Q}.$$

It is shown in [12] that the mixing angle between $\eta_{\Xi_Q}^{\text{phys}}$ and $\eta_{\Xi'_Q}^{\text{phys}}$ is equal to $\theta_1 = 6.4 \pm 1.8^0$ ($\theta_2 = 5.5 \pm 1.8^0$), which is quite small, and hence we can safely neglect it. According to sum rules method philosophy, the correlation function is calculated in terms of hadrons, and in terms of quarks and hadrons in two different kinematical regions. Equating then the obtained results to each other, one can get the sum rules for the appropriate physical quantity.

We start calculating the correlation function from the hadronic side. Inserting a complete set of hadrons with $\eta_{\Xi_Q}$ and $\eta_{\Xi'_Q}$ state quantum numbers, and isolating the ground state contributions, we get

$$\Pi = \frac{\langle 0 | \eta_{\Xi_Q} | \Xi_Q(p_1) \rangle}{(m_{\Xi_Q}^2 - p_1^2)} \frac{\langle \Xi_Q(p_1) | \eta_{\Xi'_Q} | 0 \rangle}{(m_{\Xi'_Q}^2 - p_2^2)}$$

$$\times \frac{\langle \Xi_Q(p_2) | \eta_{\Xi'_Q} | 0 \rangle}{(m_{\Xi'_Q}^2 - p_2^2)} + \cdots,$$

where $p_2 = p_1 + q$, $p_1 = p$, $q$ is the photon momentum, and dots represent the contributions of higher states and continuum. The matrix element of the interpolating current between one baryon and vacuum states is defined in the following way:

$$\langle 0 | \eta_{\Xi_Q} | \Xi_Q(p) \rangle = (\lambda_{\Xi_Q} u_{\Xi_Q}(p)),$$

$$\langle \Xi_Q(p) | \eta_{\Xi'_Q} | 0 \rangle = (\lambda_{\Xi'_Q} u_{\Xi'_Q}(p)).$$

The second matrix element in (4) is parametrized in terms of two form factors $F_1$ and $f_2$, as:

$$\langle \Xi_Q(p_1) | \eta_{\Xi'_Q} | \Xi_Q(p_2) \rangle = \bar{u}(p_1) \left[ f_1 \gamma_\mu - \frac{i \sigma_{\mu \nu} q^\nu}{m_{\Xi_Q}^2 + m_{\Xi'_Q}^2} f_2 \right] u(p_2) \epsilon^\mu,$$

$$= \bar{u}(p_1) \left[ (f_1 + f_2) \gamma_\mu - f_2 (p_1 + p_2)_\mu \right] u(p_2) \epsilon^\mu,$$

$$= \bar{u}(p_1) \left[ (f_1 + f_2) \gamma_\mu - f_2 (p_1 + p_2)_\mu \right] u(p_2) \epsilon^\mu,$$ (6)

where we set $q^2 = 0$ for the real photon. Using Eqs. (5) and (6) in the hadronic side of the correlation function, we get the following expression:

$$\Pi = \frac{\lambda_{\Xi_Q} \lambda_{\Xi'_Q}}{(m_{\Xi_Q}^2 - p_1^2)(m_{\Xi'_Q}^2 - p_2^2)} \left( (f_1 + f_2) \gamma_\mu - f_2 (p_1 + p_2)_\mu \right) \times (p_2 + m_{\Xi'_Q}) \epsilon^\mu.$$ (7)

We see from this expression that there are many structures appearing in the phenomenological part of the correlation function in calculation of the transition magnetic moment. In the present work we chose the structure $p_1 \epsilon p_2$ in determining the transition magnetic form factor $f_1 + f_2$. At $q^2 = 0$ this combination gives the transition magnetic moments in natural units, i.e., $e h / (m_{\Xi_Q}^2 + m_{\Xi'_Q}^2)$. The coefficient of the structure $p_1 \epsilon p_2$ gives the expression of the invariant function for the transition magnetic moment $\mu_{\Xi_Q \Xi'_Q}$ in the following form:

$$\Pi = \frac{\lambda_{\Xi_Q} \lambda_{\Xi'_Q}}{(m_{\Xi_Q}^2 - p_1^2)(m_{\Xi'_Q}^2 - p_2^2)} \mu_{\Xi_Q \Xi'_Q}.$$

(8)

The theoretical part of the correlation function is calculated in the following way. At the first step the correlation function is written in terms of the quark operators using the Wick theorem. The correlation function contains two parts, namely, the photon interacting with quarks perturbatively and the photon interacting with quarks nonperturbatively. In order to calculate the perturbative contribution, it is enough to replace one of the three propagators with

$$S \rightarrow - \frac{1}{2} \int dy S^{\text{free}}(x - y) \gamma_\mu S^{\text{free}}(y) \gamma_\nu \mathcal{F}_{\mu \nu},$$

and the remaining two quark propagators are taken as free quark propagators. In Eq. (9) the fixed point gauge (the so-called Fock-Schwinger gauge) is used, i.e., $A_\mu = (1/2) \gamma_\mu \mathcal{F}_{\mu \nu}$, and $S^{\text{free}}$ is the free quark operator. The free quark operator for the light and heavy quarks is given as

$$S^{\text{free}}(x) = \frac{i \gamma^\mu}{2 \pi^2 x^\mu} - \frac{m_q}{4 \pi^2 x^2},$$

$$S^{\text{free}}(x) = \frac{m_Q^2 K_1(m_Q \sqrt{-x^2})}{4 \pi^2 \sqrt{-x^2}} - \frac{i m_Q^2 K_2(m_Q \sqrt{-x^2})}{4 \pi^2 \sqrt{-x^2}},$$

(10)

respectively, where $K_k$ are the modified Bessel function of the second kind, and $m_q$ and $m_Q$ are the masses of the light
and heavy quarks, respectively. Following the above-explained steps, the perturbative part of the correlation function can be calculated straightforwardly.

The nonperturbative contribution to the correlation function can be calculated by replacing one of the light quark propagators with

$$S^{lib}_{i\bar{q}j} = -\frac{1}{4} \bar{q} \Gamma_i q^b (\Gamma_j)_{i\bar{q}j},$$

where \( \Gamma_i \) is the full set of the Dirac matrices, i.e., \( \Gamma_i = \{ 1, \gamma_5, \gamma_\mu, i\gamma_\mu \gamma_5, \sigma_\mu / \sqrt{2} \} \), and the remaining two propagators are taken as full quark propagators. So, for calculation of the perturbative and nonperturbative parts of the correlation function, we need the expressions of the light and heavy propagators in the external field.

The light cone expansion of the propagator in the external field is performed in [13], and it receives contributions from nonlocal four-quark \( \bar{q} q q q \), three-particle \( \bar{q} G q \), four-particle \( \bar{q} G G q \), etc. operators, where \( G \) is the gluon field strength tensor. In the present work we take into account contributions coming only from nonlocal operators with one gluon. The contributions of four-quark and two-gluon-two-quark operators are neglected due to their small contributions. Under these approximations the light and light quark operators, in the presence of the external field, have the following forms:

$$i S_{\text{light}}(x) = i S_{\text{free}}(x) - \frac{1}{12} \bar{q} \gamma^\mu (\Gamma_i)_{i\bar{q}j} \Gamma_j q^b \frac{1}{16} \left( 1 - \frac{m_0^2 x^2}{16} \right)$$

$$- i g_s \int_0^1 du \frac{k}{16 \pi^2 x} G_{\mu\nu}(ux) \sigma_{\mu\nu}$$

$$- i \frac{4 \pi}{x} \bar{u} v G_{\mu\nu}(ux) \gamma^\nu \gamma^\mu \gamma^\nu \gamma^\mu \gamma^\nu \gamma^\mu$$

$$i S_{\text{heavy}}(x) = i S_{\text{free}}(x) - ig_s \int \frac{d^4 k}{(2\pi)^4} e^{-ikx}$$

$$\times \int_0^1 du \frac{k + m_Q}{2(m_Q^2 - k^2)} G_{\mu\nu}(ux) \sigma_{\mu\nu}$$

$$+ \frac{u}{m_Q^2 - k^2} \bar{u} x \gamma^\mu G_{\mu\nu} \gamma^\nu \gamma^\mu$$

It follows from Eqs. (11) and (12) that in order to calculate the nonperturbative contribution to the correlation function, we need to know the following matrix elements: \( \langle \gamma(q) \bar{q} \Gamma_i q \rangle \) and \( \langle \gamma(q) \bar{q} \Gamma_i G_{\mu\nu} q \rangle \). These matrix elements are defined in terms of the photon distribution amplitudes (DAs), whose expressions all can be found in [14]. Using Eq. (12) and definitions of the photon DAs, one can calculate the theoretical part of the correlation function.

The sum rules for the magnetic moment of the \( \Xi' \Xi \) transition are obtained by equating the coefficients of the structure \( p_1 \bar{p} p_2 \) from both sides of the correlation function. The final step in deriving the sum rules for the magnetic moment of the \( \Xi' \Xi \) transition is performing double Borel transformation over the variables \( -p_1^2 \rightarrow M_1^2 \) and \( -p_2^2 = (p_1 + q)^2 \rightarrow M_2^2 \) in order to suppress higher state and continuum contributions. Finally, we get

$$\lambda_{\Xi \Xi} \lambda_{\Xi'} = e^{(m_{\Xi'}^2 - M_1^2 + M_2^2 / M_1^2)} \Pi_{\text{theor}}.$$  \( \text{(13)} \)

The expression for \( \Pi_{\text{theor}} \) is rather lengthy; therefore, we do not present it here.

It should be noted here that since the masses of \( \Xi \Xi \) and \( \Xi' \Xi \) are very close to each other, we can set \( M_1^2 = M_2^2 \), and \( m_{\Xi} = m_{\Xi'}. \)

It follows from Eq. (13) that in calculation of the transition magnetic moment, \( \mu_{\Xi \Xi} \), we need to know the residues of the \( \Xi \Xi \) and \( \Xi' \Xi \) baryons. The residues of the sextet and antitriplet heavy baryons have already been calculated in [15], and for this reason we do not present them in this work.

### III. NUMERICAL ANALYSIS

In this section we perform a numerical analysis of the sum rules for the transition magnetic moment obtained in the previous section. The main input parameters of the sum rules are photon DAs, whose explicit expressions are given in [14].

The remaining input parameters that we use in the numerical analysis of the sum rules are \( \langle \bar{q} q \rangle (1 \text{ GeV}) = -(0.243)^3 \text{ GeV}^3 \), \( \langle s s \rangle (1 \text{ GeV}) = 0.8 \langle \bar{q} q \rangle (1 \text{ GeV}) \), \( m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2 \) [16], the magnetic susceptibility \( \chi(1 \text{ GeV}) = -2.85 \pm 0.5 \text{ GeV}^{-2} \) [17]. The sum rules contain the following auxiliary parameters: Borel mass square \( M^2 \), continuum threshold \( s_{0\Lambda} \), and the arbitrary parameter \( \beta \) in the interpolating current. Obviously one expects the transition magnetic moment to be independent of these parameters. For this reason in further numerical analysis we shall look for the regions of these parameters where the magnetic moment is practically independent of them. The domain of \( M^2 \) is determined from the following two conditions:

(i) the ground state contribution should be larger compared to higher states and the continuum contributions

(ii) the highest terms in \( 1 / M^2 \) should constitute about 30%–40% of the higher power \( M^2 \) terms.

Our numerical analysis shows that both conditions are satisfied when \( 10 \leq M^2 \leq 25 \text{ GeV}^2 \) for the \( \Xi_b \) baryon, and \( 6 \leq M^2 \leq 12 \text{ GeV}^2 \) for the \( \Xi_q \) baryon, respectively. The working regions of \( s_{0\Lambda} \) and \( \beta \) are also determined from the consideration that the magnetic moment should not change appreciably in the respective regions of these parameters.

The dependence of the magnetic moments for the \( \Xi_0^0 \Xi_0^0 \), \( \Xi_0^0 \Xi_0^0 \), \( \Xi_0^0 \Xi_0^0 \), and \( \Xi_0^0 \Xi_0^0 \) transitions on \( \cos \theta \), where \( \beta = \tan \theta \), at fixed values of \( M^2 \) and \( s_{0\Lambda} \) are presented in Figs. (1)–(4), respectively. We see from these figures that the
is following values for the \( \cos \theta \) at several fixed values of the Borel mass-square \( M^2 \), and at fixed value of the continuum threshold \( s_0 = 45 \text{ GeV}^2 \).

In the case of the \( b \)-baryons, the change in the values of the magnetic moments is about 10% if \( s_0 \) varies in the domain \( 40 \leq s_0 \leq 45 \text{ GeV}^2 \). Hence, we can safely say that the results for the magnetic moments are stable with respect to the variation of \( s_0 \) in the determined domain. In Table 1 we present our results on the transition magnetic moments together with the ones predicted by different approaches, such as the effective quark mass and screened quark charge scheme (for the \( b \)-hadron sector see [18], for the \( c \)-hadron sector see [19]), bag model [20], nonrelativistic quark model [21], and chiral constituent quark model [22].

From a comparison of our results with the predictions of the above-mentioned approaches, we see that for the \( \Xi^+_c \rightarrow \Xi^+_b \) transition, all results are very close to each other. In the case of the \( \Xi^0_c \rightarrow \Xi^0_b \) transition, the results of all approaches are in good agreement with the exception of the bag model. Our prediction for the \( \Xi^0_c \rightarrow \Xi^0_b \) transition is very close to the results of [18] and [21], but considerably different from that of [20]. As far as the \( \Xi^0_c \rightarrow \Xi^0_b \) transition is concerned, our result confirms the prediction of [19]—it is approximately 1.5 times smaller than that of [22], two times

\[
\mu_{\Xi^0_c \rightarrow \Xi^0_b} = (1.4 \pm 0.1) \mu_N, \quad \mu_{\Xi^+_c \rightarrow \Xi^+_b} = (0.21 \pm 0.01) \mu_N, \quad \mu_{\Xi^0_c \rightarrow \Xi^0_b} = (1.3 \pm 0.1) \mu_N, \quad \mu_{\Xi^+_c \rightarrow \Xi^+_b} = (0.18 \pm 0.02) \mu_N,
\]

where \( \mu_N \) is the nuclear magneton. In the case of \( b \) baryons, the change in the values of the magnetic moments is about 10% if \( s_0 \) varies in the domain \( 40 \leq s_0 \leq 45 \text{ GeV}^2 \). Hence, we can safely say that the results for the magnetic moments are stable with respect to the variation of \( s_0 \) in the determined domain. In Table 1 we present our results on the transition magnetic moments together with the ones predicted by different approaches, such as the effective quark mass and screened quark charge scheme (for the \( b \)-hadron sector see [18], for the \( c \)-hadron sector see [19]), bag model [20], nonrelativistic quark model [21], and chiral constituent quark model [22].

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\[
\mu_{\Xi^0_c \rightarrow \Xi^0_b} = (1.40 \pm 0.10) 1.392 1.354 \quad \mu_{\Xi^+_c \rightarrow \Xi^+_b} = (0.21 \pm 0.01) 0.178 0.142 \quad \mu_{\Xi^0_c \rightarrow \Xi^0_b} = (1.3 \pm 0.1) -1.41 -1.39 1.043 1.4 \quad \mu_{\Xi^+_c \rightarrow \Xi^+_b} = (0.18 \pm 0.02) 0.18 0.13 0.013 0.08 -0.31
\]
larger than that of [21] and 10 times larger than the results predicted in [20].

Finally, we express our concluding remarks about the present work, in which we calculate the magnetic moments of the $\Xi_Q^{0} - \Xi_Q^{0}$ ($Q = b$ or $c$) transitions within the framework of light cone QCD sum rules. We also perform a comparison of our results with the predictions of various approaches existing in the literature. We observe that for the $\Xi_b^0 - \Xi_b^0$ and $\Xi_c^+ - \Xi_c^+$ transitions, the predictions of all approaches do practically agree with each other. A similar situation takes place for the $\Xi_b^- - \Xi_b^-$ transition, with the exception of the bag model prediction. As it comes to the $\Xi_Q^{0} - \Xi_Q^{0}$ transition, our results coincide with the ones predicted in [16], but differ substantially with the predictions of all other approaches. At present, these magnetic moments have not yet been measured in experiments, and we hope that they all can be measured in future planned experiments. We believe that the measurement of $\Xi_Q^{0} - \Xi_Q^{0}$ transition magnetic moments will also be very useful in the determination of the mixing angle among heavy baryons.