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Resonance X(5568) as an exotic axial-vector state

S.S. Agaev[1], K. Azizi[2,a], B. Barsbay[3], and H. Sundu[3]

[1] Institute for Physical Problems, Baku State University, Az–1148 Baku, Azerbaijan
[2] Department of Physics, Doğuş University, Acibadem-Kadiköy, 34722 Istanbul, Turkey
[3] Department of Physics, Kocaeli University, 41380 Izmit, Turkey

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Abstract. The mass and meson-current coupling constant of the resonance $X(5568)$, as well as the width of the decay $X(5568) \to B_s^*\pi$ are calculated by modeling the exotic $X(5568)$ resonance as a diquark-antidiquark state $X_b = [su][bd]$ with quantum numbers $J^P = 1^+$. The calculations are made employing QCD two-point sum rule method, where the quark, gluon and mixed vacuum condensates up to dimension eight are taken into account. The sum rule approach on the light-cone in its soft-meson approximation is used to explore the vertex $X_b B_s^* \pi$ and extract the strong coupling $g_{X_b B_s^* \pi}$, which is a necessary ingredient to find the width of the $X_b \to B_s^* \pi^+$ decay process. The obtained predictions are compared with the experimental data of the D0 Collaboration, and results of other theoretical works.

1 Introduction

Investigation of the "exotic" hadrons, which cannot be described as usual $q\bar{q}$ and qqq structures, and are composed of more than three valence quarks is now one of the intriguing and developing branches of high energy physics. The existence of such particles, their possible properties, and experiments suitable for their observation were among the interests of physicists during the last three decades. Starting from the discovery of the theory of strong interactions, *i.e.* the Quantum Chromodynamics (QCD), a qualitative analysis of the exotic states was replaced by the quantitative calculations of their parameters in the strong context of the quantum field theory. Some of the parameters of the exotic states were computed in the eighties using namely the nonperturbative tools of QCD in refs. [1–5]. But theoretical studies of the exotic particles then were not supported by a reliable experimental information, which slowed the growth of the field.

The situation changed when the various collaborations, such as Belle, BaBar, BESIII, LHCb, CDF, and D0 started to supply hadron physics with valuable experimental data on the quantum numbers, masses, and decay widths of new heavy resonances, some of which were interpreted as four-quark (*i.e.*, as tetraquark) exotic states [6–9]. Experimental studies, naturally, boosted suggestion of new theoretical models to explain an internal structure of the observed resonances, as well as led to the invention of new or adapting existing nonperturbative approaches to meet problems of the new emerging field of hadron physics. It is worth noting that during the last years a considerable success was achieved both in the experimental and theoretical studies of the exotic states (see, the reviews [10–18] and references therein).

Among the family of the tetraquarks the narrow resonance $X(5568)$ has a unique position. The evidence for the existence of this resonance was announced recently by the D0 Collaboration [19], which was founded on the $p\bar{p}$ collision data at $\sqrt{s} = 1.96\,\text{TeV}$. It was observed in the channel $X(5568) \to B_s^0\pi^\pm$ through the chain of decays $B_s^0 \to J/\psi\phi$, $J/\psi \to \mu^+\mu^-$, and $\phi \to K^+K^-$. It is not difficult to conclude that the state $X(5568)$ consists of valence b, s, u and d quarks, and is presumably the first observed particle built of four different quarks. The measured mass of this state is $m_X = 5567.8 \pm 2.9(\text{stat})^{+0.9}_{-1.9}(\text{syst})\,\text{MeV}$, and its decay width is $\Gamma = 21.9 \pm 6.4(\text{stat})^{+5.0}_{-2.5}(\text{syst})\,\text{MeV}$. The D0 assigned to this particle the quantum numbers $J^P = 0^+$. But if the resonance $X(5568)$ decays as $X(5568) \to B_s^*\pi^\pm \to B_s^0\gamma\pi^\pm$ with unseen soft γ, then the difference $m(B_s^*) - m(B_s^0)$ should be added to the mass of the resonance $X(5568)$, whereas its decay width remains unchanged [19]. In this case the resonance has quantum numbers $J^P = 1^+$. Stated differently, in accordance with the results of the D0 Collaboration, the resonance $X(5568)$ may be treated as the particle with $J^P = 0^+$ or $J^P = 1^+$.

The decay channel $B_s^0\pi^\pm$ was investigated by the LHCb Collaboration utilizing the pp collision data at energies of 7 TeV and 8 TeV collected at CERN [20]. The

[a] e-mail: kazizi@dogus.edu.tr

aim was to confirm the existence of the $X(5568)$ state and measure its spectroscopic parameters. But the LHCb Collaboration could not fix the resonance structure in the $B_s^0\pi^\pm$ invariant mass distribution at energies less than 6000 MeV. Very similar conclusions were drawn by the CMS Collaboration [21], in which a mass range up to 5900 MeV was searched for a possible structure, setting an upper limit. Nevertheless, the D0 Collaboration from the analysis of the semileptonic decays of B_s^0 meson recently confirmed in ref. [22] the observation of the $X(5568)$ resonance. As is seen, the experimental situation with the existence of the exotic state $X(5568)$, supposedly built of four different quark flavors, remains intriguing and unclear.

Namely these conditions make the theoretical studies of the $X(5568)$ state even more urgent than just after the first announcement of its observation. Suggestions about the inner organization of this particle as the bound state of a diquark and antidiquark, or as a meson molecule compound were already made in ref. [19]. Theoretical works, appeared afterwards to determine the mass and decay width of the $X(5568)$ state, followed mainly these suggestions. Thus, in ref. [23] it was considered as the diquark-antidiquark structure $X_b = [su][\bar{b}\bar{d}]$ with the quantum numbers 0^{++}, where its mass m_{X_b} and meson-current coupling (hereafter, the meson coupling) f_{X_b} were calculated. In the framework of the diquark-antidiquark model the mass and other parameters of $X(5568)$ were also explored in refs. [24–27]. The values for m_{X_b} found in these works agree with each other, and are consistent with the experimental data of the D0 Collaboration.

The width of the $X_b \to B_s^0\pi^+$ decay channel was calculated in ref. [28] employing for X_b the same structure and interpolating current as in ref. [23]. To this end, authors applied the QCD light-cone sum rule method and soft-meson approximation adjusted for studying of the tetraquark states in ref. [29], where relevant explanations and technical details can be found. The result for $\Gamma(X_b^+ \to B_s^0\pi^+)$ derived in ref. [28] describes correctly the experimental data. The width of the decay channels $X^\pm(5568) \to B_s\pi^\pm$ was also analyzed in refs. [30,31] employing the three-point QCD sum rule approach. In these works authors found a nice agreement between the theoretical predictions for $\Gamma(X^\pm \to B_s^0\pi^\pm)$ and data, as well.

As we have mentioned above, the $X(5568)$ state can also be considered as a molecule composed of B and $\overline{K}$ mesons, which was realized in refs. [32–34]. But in this scenario, in accordance with ref. [33], the mass of such molecule and width of the decay $X^\pm(5568) \to B_s\pi^\pm$ exceed the experimental data of D0, and predictions of the diquark-antidiquark model. These facts were interpreted in favor of the diquark-antidiquark organization of the $X(5568)$ state.

The experimental data of the D0 and LHCb Collaborations generated the appearance of interesting theoretical works, where the structure and spectroscopic parameters, production mechanisms of the $X(5568)$ state were considered. The details of the used methods, necessary explanations, and conclusions made there concerning the nature of the resonance $X(5568)$ can be found in the original papers [35–47].

In the present work we calculate the mass, meson coupling of the $X_b = [su][\bar{b}\bar{d}]$ diquark-antidiquark state treating it as an axial-vector particle with the quantum numbers $J^P = 1^+$. To this end, we apply the QCD two-point sum rule approach and include into our analysis the quark, gluon and mixed condensates up to eight dimensions. We are also going to determine the width of the decay $X_b \to B_s^*\pi^+$ using the soft-meson version of the QCD light-cone sum rule method. Performed investigation should allow us to answer a question: is the $X(5568)$ resonance a state with $J^P = 0^+$ or $J^P = 1^+$?

This work is organized in the following form. Section 2 is devoted to the sum rule calculations of the mass and meson coupling of the axial vector X_b state. Here we derive also the light-cone sum rule expression for the strong coupling $g_{X_bB_s^*\pi}$ necessary to compute the decay width of the process $X_b \to B_s^*\pi^+$. In sect. 3 we conduct numerical calculations and extract values of the mass, meson coupling and decay width under consideration. Here we compare our results with the experimental data and predictions of other theoretical works. Section 4 contains our brief conclusions on the nature of the X_b state based on the present studies. The explicit expressions for the spectral density used in the two-point sum rules are given in the appendix.

2 The QCD sum rules for the parameters of the axial-vector X_b state

In this section we derive QCD sum rules required for both the mass and decay width calculations. To this end, we calculate the two-point spectral density necessary for the mass and meson coupling computations. To extract the width of the decay channel $X_b \to B_s^*\pi^+$, we find the strong coupling $g_{X_bB_s^*\pi}$ using the spectral densities corresponding to different Lorentz structures in the corresponding correlation function.

2.1 Sum rules for the mass and meson coupling

In order to find QCD two-point sum rules for the calculation of the mass and meson coupling of the X_b state we consider the correlation function given below as

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle 0|\mathcal{T}\{J_\mu(x)J_\nu^\dagger(0)\}|0\rangle, \qquad (1)$$

where $J_\mu(x)$ is the interpolating current with the quantum numbers of the X_b state. We consider X_b as a particle with the quantum numbers $J^P = 1^+$. Then in the diquark-antidiquark model one of the acceptable interpolating currents $J_\mu(x)$ is defined by the expression [26]

$$J_\mu(x) = s_a^T(x)C\gamma_5 u_b(x)\left\{\bar{b}_a(x)\gamma_\mu C\bar{d}_b^T(x) - \bar{b}_b(x)\gamma_\mu C\bar{d}_a^T(x)\right\}, \qquad (2)$$

where a and b are color indices and C is the charge conjugation matrix.

In order to derive the QCD sum rule expression we first have to calculate the correlation function in terms of the physical parameters of the X_b state. Saturating the correlation function with a complete set of the X_b state and performing an integral over x in eq. (1), we get

$$\Pi_{\mu\nu}^{\rm Phys}(q)=\frac{\langle 0|J_\mu|X_b(q)\rangle\langle X_b(q)|J_\nu^\dagger|0\rangle}{m_{X_b}^2-q^2}+\ldots \tag{3}$$

with m_{X_b} being the mass of the X_b state. Here the dots indicate contributions to the correlation function arising from the higher resonances and continuum states. We define the meson coupling f_{X_b} using the matrix element

$$\langle 0|J_\mu|X_b(q)\rangle=f_{X_b}m_{X_b}\varepsilon_\mu, \tag{4}$$

where ε_μ is the polarization vector of the X_b state. Then in terms of m_{X_b} and f_{X_b}, the correlation function can be written in the form

$$\Pi_{\mu\nu}^{\rm Phys}(q)=\frac{m_{X_b}^2f_{X_b}^2}{m_{X_b}^2-q^2}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{m_{X_b}^2}\right)+\ldots. \tag{5}$$

Equation (5) has been derived within the single pole assumption, which in the case of a multiquark state requires additional justification. The reason is that for such systems in the physical side of the sum rule one has to take into account also two-hadron reducible contributions, which for the tetraquarks however are negligible [48]. Therefore, we can apply the Borel transformation directly to eq. (5), which yields

$$\mathcal{B}_{q^2}\Pi_{\mu\nu}^{\rm Phys}(q)=m_{X_b}^2f_{X_b}^2e^{-m_{X_b}^2/M^2}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{m_{X_b}^2}\right)+\ldots. \tag{6}$$

From the QCD side the same function must be calculated employing the quark-gluon degrees of freedom. To this end, we contract the heavy and light quark fields and find for the correlation function $\Pi_{\mu\nu}^{\rm QCD}(q)$ the following expression:

$$\begin{aligned}\Pi_{\mu\nu}^{\rm QCD}(q)=i\int d^4xe^{iqx}\Big\{&{\rm Tr}\left[\gamma_5\widetilde{S}_s^{aa'}(x)\right.\\&\left.\times\gamma_5S_u^{bb'}(x)\right]{\rm Tr}\left[\gamma_\mu\widetilde{S}_d^{a'b}(-x)\gamma_\nu S_b^{b'a}(-x)\right]\\&-{\rm Tr}\left[\gamma_\mu\widetilde{S}_d^{b'b}(-x)\gamma_\nu S_b^{a'a}(-x)\right]{\rm Tr}\left[\gamma_5\widetilde{S}_s^{aa'}(x)\right.\\&\left.\times\gamma_5S_u^{bb'}(x)\right]+{\rm Tr}\left[\gamma_5\widetilde{S}_s^{aa'}(x)\gamma_5S_u^{bb'}(x)\right]\\&\times{\rm Tr}\left[\gamma_\mu\widetilde{S}_d^{b'a}(-x)\gamma_\nu S_b^{a'b}(-x)\right]\\&-{\rm Tr}\left[\gamma_\mu\widetilde{S}_d^{a'a}(-x)\gamma_\nu S_b^{b'b}(-x)\right]\\&\times{\rm Tr}\left[\gamma_5\widetilde{S}_s^{aa'}(x)\gamma_5S_u^{bb'}(x)\right]\Big\}. \end{aligned}\tag{7}$$

In eq. (7) $S_{s(u,d)}^{ab}(x)$ and $S_b^{ab}(x)$ are the light s, u, d quarks and b-quark propagators, respectively. Here we also use the notation

$$\widetilde{S}_{s(d)}(x)=CS_{s(d)}^T(x)C.$$

We work with the light quark propagator $S_q^{ab}(x)$ defined in the form

$$\begin{aligned}S_q^{ab}(x)=&i\delta_{ab}\frac{\not x}{2\pi^2x^4}-\delta_{ab}\frac{m_q}{4\pi^2x^2}-\delta_{ab}\frac{\langle\overline{q}q\rangle}{12}\\&+i\delta_{ab}\frac{\not x m_q\langle\overline{q}q\rangle}{48}-\delta_{ab}\frac{x^2}{192}\langle\overline{q}g_s\sigma Gq\rangle+i\delta_{ab}\frac{x^2\not x m_q}{1152}\\&\times\langle\overline{q}g_s\sigma Gq\rangle-i\frac{g_sG_{ab}^{\alpha\beta}}{32\pi^2x^2}\left[\not x\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\not x\right]\\&-i\delta_{ab}\frac{x^2\not x g_s^2\langle\overline{q}q\rangle^2}{7776}-\delta_{ab}\frac{x^4\langle\overline{q}q\rangle\langle g_s^2G^2\rangle}{27648}+\ldots.\end{aligned}\tag{8}$$

Let us emphasize that in the calculations we set the light quark masses m_u and m_d equal to zero, preserving at the same time the dependence of the propagator $S_s^{ab}(x)$ on the m_s. For the b-quark propagator $S_b^{ab}(x)$ we employ the formula given in ref. [49]

$$\begin{aligned}S_b^{ab}(x)=&i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\left\{\frac{\delta_{ab}(\not k+m_b)}{k^2-m_b^2}\right.\\&-\frac{g_sG_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not k+m_b)+(\not k+m_b)\sigma_{\alpha\beta}}{(k^2-m_b^2)^2}\\&+\frac{g_s^2G^2}{12}\delta_{ab}m_b\frac{k^2+m_b\not k}{(k^2-m_b^2)^4}+\frac{g_s^3G^3}{48}\delta_{ab}\frac{(\not k+m_b)}{(k^2-m_b^2)^6}\\&\left.\times\left[\not k\left(k^2-3m_b^2\right)+2m_b\left(2k^2-m_b^2\right)\right](\not k+m_b)+\ldots\right\}.\end{aligned}\tag{9}$$

In eqs. (8) and (9) we use the notations

$$\begin{aligned}G_{ab}^{\alpha\beta}&=G_A^{\alpha\beta}t_{ab}^A,\qquad G^2=G_{\alpha\beta}^AG_{\alpha\beta}^A,\\G^3&=f^{ABC}G_{\mu\nu}^AG_{\nu\delta}^BG_{\delta\mu}^C,\end{aligned}\tag{10}$$

where $a,b=1,2,3$ and $A,B,C=1,2,\ldots,8$ are the color indices. Here $t^A=\lambda^A/2$, and λ^A are the Gell-Mann matrices. In the nonperturbative terms the gluon field strength tensor $G_{\alpha\beta}^A\equiv G_{\alpha\beta}^A(0)$ is fixed at $x=0$.

The correlation function $\Pi_{\mu\nu}^{\rm QCD}(q)$ can be decomposed over the Lorentz structures $\sim g_{\mu\nu}$ and $\sim q_\mu q_\nu$. The QCD sum rule expressions can be obtained after fixing the same Lorentz structures in both $\Pi_{\mu\nu}^{\rm Phys}(q)$ and $\Pi_{\mu\nu}^{\rm QCD}(q)$. We choose the term $\sim g_{\mu\nu}$, which receives a contribution from only spin-1 states, whereas the invariant amplitude corresponding to the structure $\sim q_\mu q_\nu$ forms due to contributions of both spin-0 and spin-1 states.

The chosen invariant amplitude $\Pi^{\rm QCD}(q^2)$ can be written down as the dispersion integral

$$\Pi^{\rm QCD}(q^2)=\int_{(m_b+m_s)^2}^\infty\frac{\rho^{\rm QCD}(s)}{s-q^2}ds+\ldots, \tag{11}$$

where $\rho^{\rm QCD}(s)$ is the two-point spectral density. Now utilizing the Borel transformation to $\Pi^{\rm QCD}(q^2)$, equating the obtained expression with the relevant part of the function $\mathcal{B}_{q^2}\Pi_{\mu\nu}^{\rm Phys}(q)$, and subtracting the continuum contribution we, as a result, get the required sum rule. Then the mass of the X_b state can be evaluated from the sum rule

$$m_{X_b}^2 = \frac{\int_{(m_b+m_s)^2}^{s_0} \mathrm{d}s s\rho^{\mathrm{QCD}}(s)e^{-s/M^2}}{\int_{(m_b+m_s)^2}^{s_0} \mathrm{d}s\rho(s)e^{-s/M^2}}. \qquad (12)$$

To extract the meson coupling f_{X_b} we can employ the sum rule formula

$$m_{X_b}^2 f_{X_b}^2 e^{-m_{X_b}^2/M^2} = \int_{(m_b+m_s)^2}^{s_0} \mathrm{d}s\rho^{\mathrm{QCD}}(s)e^{-s/M^2}. \qquad (13)$$

The key component of these expressions is the two-point spectral density $\rho^{\mathrm{QCD}}(s)$. Because the methods for its derivation were presented in the literature (see, ref. [29]), here we omit technical details of calculations moving the final expressions of the spectral density and its components to the appendix.

2.2 Strong coupling $g_{X_b B_s^* \pi}$ and width of the decay $X_b \to B_s^* \pi^+$

In this subsection we outline the soft-meson approximation of the QCD light-cone sum rule method used here to explore the strong vertex $X_b B_s^* \pi$ and find the sum rule expression for the coupling $g_{X_b B_s^* \pi}$. The latter will be used to calculate the width of the $X_b \to B_s^* \pi^+$ decay process.

To this end, we start from the correlation function

$$\Pi_{\mu\nu}(p,q) = i\int \mathrm{d}^4 x e^{ipx}\langle \pi(q)|\mathcal{T}\{J_\mu^{B_s^*}(x)J_\nu^\dagger(0)\}|0\rangle, \qquad (14)$$

where the interpolating current for the B_s^* meson has the form

$$J_\mu^{B_s^*}(x) = \bar{b}_l(x)\gamma_\mu s_l(x), \qquad (15)$$

whereas $J_\nu(x)$ is defined by eq. (2). Here p, q and $p' = p+q$ are the momenta of the mesons B_s^* and π, and the X_b state, respectively.

To derive the sum rule for the strong coupling $g_{X_b B_s^* \pi}$, we follow the standard prescriptions of the QCD sum rule approach and calculate $\Pi_{\mu\nu}(p,q)$ in terms of the physical parameters of the involving particles. Then we obtain

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(p,q) = \frac{\langle 0|J_\mu^{B_s^*}|B_s^*(p)\rangle}{p^2 - m_{B_s^*}^2}\langle B_s^*(p)\pi(q)|X_b(p')\rangle \times \frac{\langle X_b(p')|J_\nu^\dagger|0\rangle}{p'^2 - m_{X_b}^2} + \dots . \qquad (16)$$

where the dots denote, as usual, contributions of the higher resonances and continuum states. The expression of the function $\Pi_{\mu\nu}^{\mathrm{Phys}}(p,q)$ can be further simplified and expressed in terms of the particle parameters if we introduce the new matrix elements

$$\langle 0|J_\mu^{B_s^*}|B_s^*(p)\rangle = f_{B_s^*} m_{B_s^*}\varepsilon_\mu,$$
$$\langle B_s^*(p)\pi(q)|X_b(p')\rangle = [(p\cdot p')(\varepsilon^*\cdot\varepsilon') - (p\cdot\varepsilon')(p'\cdot\varepsilon^*)]\, g_{X_b B_s^* \pi}, \qquad (17)$$

where $f_{B_s^*}$, $m_{B_s^*}$, ε_μ are the decay constant, mass and polarization vector of the $B_s^*(p)$ meson, and ε'_ν is the polarization vector of the X_b state.

Using these matrix elements, as well as that given by eq. (4), one can rewrite the correlation function as

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(p,q) = \frac{f_{B_s^*} f_{X_b} m_{X_b} m_{B_s^*}}{(p'^2 - m_{X_b}^2)(p^2 - m_{B_s^*}^2)} g_{X_b B_s^* \pi} \times \left(\frac{m_{X_b}^2 + m_{B_s^*}^2}{2} g_{\mu\nu} - p'_\mu p_\nu\right) + \dots . \qquad (18)$$

It evidently contains two Lorentz structures $g_{\mu\nu}$ and $p'_\mu p_\nu$. We are going to use both of them to derive the sum rules for the coupling $g_{X_b B_s^* \pi}$ and compare our results with each other in order to estimate the sensitivity of the obtained predictions to the chosen structures. Below we consider explicitly the correlation function corresponding to $g_{\mu\nu}$, and provide only the final expression for the spectral density $\widetilde{\rho}_c^{\mathrm{QCD}}(s)$ derived using the terms $\sim p'_\mu p_\nu$.

In the soft-meson limit, *i.e.* in the limit $q = 0$, the Borel transformation of the invariant amplitude corresponding to $g_{\mu\nu}$ has the form (ref. [29])

$$\Pi^{\mathrm{Phys}}(M^2) = f_{B_s^*} f_{X_b} m_{X_b} m_{B_s^*} g_{X_b B_s^* \pi} m^2 \times \frac{1}{M^2} e^{-m^2/M^2} + \cdots , \qquad (19)$$

where $m^2 = (m_{X_b}^2 + m_{B_s^*}^2)/2$.

It is known that, to obtain the sum rule expression for the strong coupling of the three-hadron vertex in the soft-meson approximation, one has to apply the one-variable Borel transformation instead of the two-variable Borel transformation accepted in the standard light-cone sum rule procedures [29]. As a result, the hadronic part of the sum rule becomes more complicated and contains contributions of transitions from exited states to the ground level, shown in eq. (19) as the dots, which in the soft-meson approximation are unsuppressed even after the Borel transformation. In order to remove these terms from the hadronic side one has to use some technical tools. One of such methods was suggested in ref. [50], which implies acting by the operator

$$\left(1 - M^2 \frac{\mathrm{d}}{\mathrm{d}M^2}\right) M^2 e^{m^2/M^2} \qquad (20)$$

to both the hadronic and QCD sides of the sum rule. It was successfully used in ref. [29], and is adopted in the present work, as well.

But before that we have to calculate the correlation function eq. (14) in terms of the quark propagators and find the QCD side of the sum rule. Contractions of s- and b-quark fields in eq. (14) give

$$\Pi_{\mu\nu}^{\mathrm{QCD}}(p,q) = i\int \mathrm{d}^4 x e^{ipx}\left\{ \left[\gamma_5 \widetilde{S}_s^{ia}(x)\gamma_\mu \times \widetilde{S}_b^{bi}(-x)\gamma_\nu\right]_{\alpha\beta} \langle \pi(q)|\overline{u}_\alpha^b d_\beta^a|0\rangle - \left[\gamma_5 \widetilde{S}_s^{ia}(x)\gamma_\mu \widetilde{S}_b^{ai}(-x)\gamma_\nu\right]_{\alpha\beta} \langle \pi(q)|\overline{u}_\alpha^b d_\beta^b|0\rangle \right\}, \qquad (21)$$

where α and β are the spinor indices, the quark fields are defined at $x = 0$.

In the soft-meson approximation the QCD side of the sum rule is considerably simpler than the one in the standard approach. Indeed, in the standard light-cone sum rule the nonlocal matrix elements of the hadrons (in the case under analysis, of the pion) are expressed in terms of their various distribution amplitudes, whereas in the soft-meson limit we get only few local matrix elements.

The following stages of calculations include the application of the expansion

$$\overline{u}^b_\alpha d^a_\beta \to \frac{1}{4}\Gamma^j_{\beta\alpha}\left(\overline{u}^b \Gamma^j d^a\right), \tag{22}$$

where Γ^j is the full set of Dirac matrixes, performing the color summation and inserting into the quark matrix elements the gluon field strength tensor G. These operations lead to the correlation function $\Pi^{\mathrm{QCD}}_{\mu\nu}(p,q)$, which depends on the two- and three-particle matrix elements of the pion: Let us note that in the present work we neglect terms $\sim G^2$ and $\sim G^3$. The final step in this way is the computation of the imaginary part of the obtained correlation function to extract the desired spectral density. Because these manipulations are described in a clear form in ref. [29], we refrain from providing further details and write down the final expression for $\rho^{\mathrm{QCD}}_c(s)$ obtained in this work as the sum of its perturbative and nonperturbative parts

$$\rho^{\mathrm{QCD}}_c(s) = \rho^{\mathrm{pert.}}_c(s) + \rho^{\mathrm{n.\text{-}pert.}}_c(s), \tag{23}$$

where

$$\rho^{\mathrm{pert.}}_c(s) = \frac{f_\pi \mu_\pi}{48\pi^2 s^2}(m_b^2 - s)(m_b^4 + m_b^2 s - 6 m_b m_s s - 2s^2) \tag{24}$$

and

$$\begin{aligned}\rho^{\mathrm{n.\text{-}pert.}}_c(s) = {}& \frac{f_\pi \mu_\pi}{12}\langle \overline{s}s\rangle \Big[s m_s \delta^{(1)}(s - m_b^2) \\ & + (m_s - 2m_b)\delta(s - m_b^2)\Big] + \frac{f_\pi\mu_\pi}{72}\langle \overline{s} g_s \sigma G s\rangle \\ & \times \Big\{ 3(2m_b - m_s)\delta^{(1)}(s - m_b^2) + s(3m_b - 5m_s) \\ & \times \delta^{(2)}(s - m_b^2) - s^2 m_s \delta^{(3)}(s - m_b^2)\Big\}.\end{aligned} \tag{25}$$

The contributions $\sim \delta^{(n)}(s - m_b^2) = (d/ds)^n \delta(s - m_b^2)$ arise from the imaginary part of the pole terms.

Calculations of the spectral density $\widetilde{\rho}^{\mathrm{QCD}}_c(s)$ performed employing the terms which in the soft limit transform to ones $\sim p'_\mu p_\nu$ yield:

$$\widetilde{\rho}^{\mathrm{pert.}}_c(s) = \frac{f_\pi\mu_\pi}{24\pi^2 s^3}(s^3 - 3 s m_b^4 + 2 m_b^6) \tag{26}$$

and

$$\begin{aligned}\widetilde{\rho}^{\mathrm{n.\text{-}pert.}}_c(s) = {}& -\frac{f_\pi\mu_\pi}{6} m_s \langle \overline{s}s\rangle \delta^{(1)}(s - m_b^2) \\ & + \frac{f_\pi\mu_\pi}{36} m_s \langle \overline{s} g_s \sigma G s\rangle \Big[4\delta^{(2)}(s - m_b^2) \\ & + s\delta^{(3)}(s - m_b^2)\Big].\end{aligned} \tag{27}$$

As is seen, the spectral densities depend on the parameters f_π and μ_π of the pion through the local matrix element

$$\langle 0|\overline{d} i\gamma_5 u|\pi(q)\rangle = f_\pi \mu_\pi, \tag{28}$$

where

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = -\frac{2\langle \overline{q}q\rangle}{f_\pi^2}. \tag{29}$$

Performing the continuum subtraction in the standard manner and applying the operator eq. (20), we get the sum rule to evaluate the strong coupling

$$\begin{aligned} g_{X_b B_s^*\pi} = {}& \frac{1}{f_{B_s^*} f_{X_b} m_{X_b} m_{B_s^*} m^2}\left(1 - M^2 \frac{\mathrm{d}}{\mathrm{d}M^2}\right) M^2 \\ & \times \int_{(m_b + m_s)^2}^{s_0} \mathrm{d}s e^{(m^2 - s)/M^2} \rho^{\mathrm{QCD}}_c(s).\end{aligned} \tag{30}$$

Similar computations for the structure $p'_\mu p_\nu$ give

$$\begin{aligned} \widetilde{g}_{X_b B_s^*\pi} = {}& \frac{1}{f_{B_s^*} f_{X_b} m_{X_b} m_{B_s^*}}\left(1 - M^2 \frac{\mathrm{d}}{\mathrm{d}M^2}\right) M^2 \\ & \times \int_{(m_b + m_s)^2}^{s_0} \mathrm{d}s e^{(m^2 - s)/M^2} \widetilde{\rho}^{\mathrm{QCD}}_c(s).\end{aligned} \tag{31}$$

The width of the decay $X_b \to B_s^* \pi^+$ can be borrowed from ref. [29], and is identical for both the strong couplings $g_{X_b B_s^*\pi}$ and $\widetilde{g}_{X_b B_s^*\pi}$:

$$\begin{aligned}\Gamma\left(X_b \to B_s^*\pi^+\right) = {}& \frac{g^2_{X_b B_s^*\pi} m^2_{B_s^*}}{24\pi}\lambda\left(m_{X_b}, m_{B_s^*}, m_\pi\right) \\ & \times \left[3 + \frac{2\lambda^2\left(m_{X_b}, m_{B_s^*}, m_\pi\right)}{m^2_{B_s^*}}\right],\end{aligned} \tag{32}$$

where

$$\lambda(a,b,c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2(a^2 b^2 + a^2 c^2 + b^2 c^2)}}{2a}.$$

The final expressions (30), (31) and (32) will be used for the numerical analysis of the decay channel $X_b \to B_s^*\pi^+$.

3 Numerical results

The QCD sum rules contain numerous parameters, *i.e.* quark, gluon and mixed condensates, masses of the b and s quarks, and B_s^* meson's mass and decay constant $f_{B_s^*}$. Values of these parameters are written down in table 1. Let us note that we use the well-known values for the quark-gluon condensates and fix them within usual procedures. The gluon condensate $\langle g_s^3 G^3\rangle$, which is not commonly employed in the sum rule calculations, is borrowed from ref. [51]. Further we choose the mass of the b quark in the $\overline{MS}$ scheme at the scale $\mu = m_b$, whereas for the decay constant $f_{B_s^*}$ invoke the result presented in ref. [52]. Other parameters are taken from ref. [53].

The QCD sum rule expressions depend also on the continuum threshold s_0 and Borel parameter M^2. To extract

Table 1. Input parameters.

Parameters	Values
$m_{B_s^*}$	$(5415.4^{+2.4}_{-2.1})$ MeV
$m_{B_s^0}$	(5366.77 ± 0.24) MeV
$f_{B_s^*}$	(225 ± 9) MeV
m_π	139.57 MeV
f_π	0.131 GeV
m_b	(4.18 ± 0.03) GeV
m_s	(95 ± 5) MeV
$\langle \bar{q}q \rangle$	$(-0.24 \pm 0.01)^3$ GeV^3
$\langle \bar{s}s \rangle$	$0.8\ \langle \bar{q}q \rangle$
m_0^2	(0.8 ± 0.1) GeV^2
$\langle \bar{s}g_s\sigma Gs \rangle$	$m_0^2 \langle \bar{s}s \rangle$
$\langle \frac{\alpha_s G^2}{\pi} \rangle$	(0.012 ± 0.004) GeV^4
$\langle g_s^3 G^3 \rangle$	(0.57 ± 0.29) GeV^6

values of the quantities under consideration we have to choose such regions for these parameters, where the dependence of the physical quantities under consideration on them is minimal. In practice, however we may only reduce the effect connected with our choices of the windows for s_0 and M^2.

The QCD sum rule method, as we know, suffers from theoretical uncertainties, which are its unavoidable property. The main sources of ambiguities in extracting the physical quantities under question are the continuum threshold s_0 and Borel parameter M^2. But procedures to extract these parameters are well defined in the context of the sum rule method itself, where the obtained results depend on the accuracy of the calculations.

Here some comments are necessary concerning the theoretical accuracy achieved in the present work in deriving the spectral density $\rho^{\mathrm{QCD}}(s)$. In fact, there are some features of $\rho^{\mathrm{QCD}}(s)$, which distinguish it from the existing calculations. First, all its higher dimensional components $\rho_k(s)$, including $\rho_7(s)$ one, are nonzero and contribute to $\rho^{\mathrm{QCD}}(s)$. Secondly, apart from the quark, gluon and mixed condensates $\langle \bar{q}q \rangle$, $\langle \alpha_s G^2/\pi \rangle$ and $\langle \bar{q}g_s\sigma Gq \rangle$, the spectral density $\rho^{\mathrm{QCD}}(s)$ encompasses effects of the terms $\sim \langle \alpha_s G^2/\pi \rangle^2$ and $\sim \langle g_s^3 G^3 \rangle$ appearing due to more detailed formulas for the quark propagators accepted in the present work.

The working window for the Borel parameter is determined from the joint requirement of the ground state dominance in the sum rule and the convergence of the relevant operator product expansion. The latter implies the suppression of the nonperturbative terms' contributions to the sum rule within the chosen interval for M^2. As a result, for the mass and meson coupling calculations we fix the following range for M^2:

$$3\,\mathrm{GeV}^2 \leq M^2 \leq 6\,\mathrm{GeV}^2. \tag{33}$$

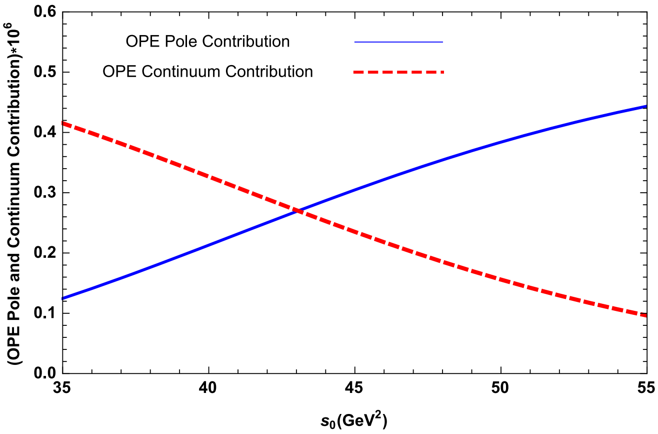

Fig. 1. The pole/continuum contribution to the sum rule. By the red (dashed) curve the total higher states and continuum contributions are plotted.

The choice of the continuum threshold s_0 depends on the energy of the first excited state with the same quantum numbers and content as the particle under consideration. It can be extracted from the comparative analysis of the pole contribution and higher states continuum contributions in the relevant operator product expansion: The former contribution should overcome the latter one. Results of the performed numerical computations are shown in fig. 1.

The analysis performed on the basis of this $\rho^{\mathrm{QCD}}(s)$ allows us to determine the range of s_0 as

$$43\,\mathrm{GeV}^2 \leq s_0 \leq 45\,\mathrm{GeV}^2. \tag{34}$$

With these main parameters at hands we can proceed and carry out computations of the mass and meson coupling of the X_b state utilizing corresponding two-point sum rules. Our results for m_{X_b} and f_{X_b} are plotted in figs. 2 and 3. As is seen, in the working regions of s_0 and M^2 the mass and meson coupling demonstrate dependences on these parameters, which are, nevertheless, mild. Hence, considering X_b as the axial-vector diquark-antidiquark state, for its mass we obtain

$$m_{X_b} = (5864 \pm 158)\,\mathrm{MeV}, \tag{35}$$

whereas for the meson-current coupling f_{X_b} we get

$$f_{X_b} = (0.42 \pm 0.14) \cdot 10^{-2}\,\mathrm{GeV}^4. \tag{36}$$

To compare our result for the mass of the state X_b with quantum numbers $J^P = 1^+$ with the experimental data of the D0 Collaboration, we first have to find this value. In accordance with explanations in ref. [19], it is defined by the expression

$$m[X(1^+)] = m[X(0^+)] + m(B_s^*) - m(B_s^0). \tag{37}$$

This equality, when taking into account $m(B_s^*) - m(B_s^0) \simeq$ 48.7 MeV (see, ref. [53]), leads to the experimental value

$$m[X(1^+)] \simeq 5617\,\mathrm{MeV}. \tag{38}$$

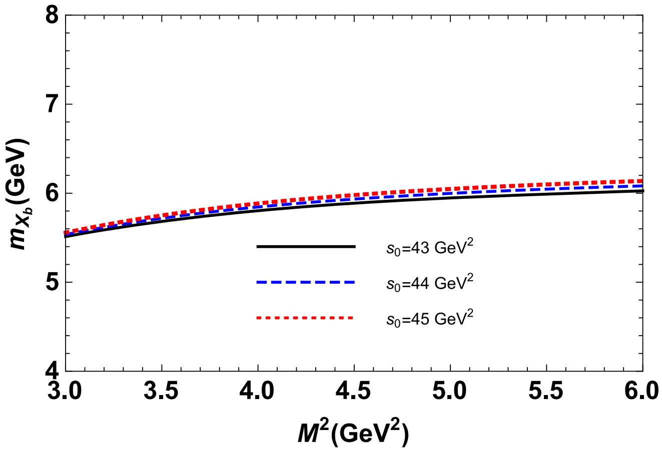

Fig. 2. The mass m_{X_b} of the axial-vector X_b state *vs.* Borel parameter M^2.

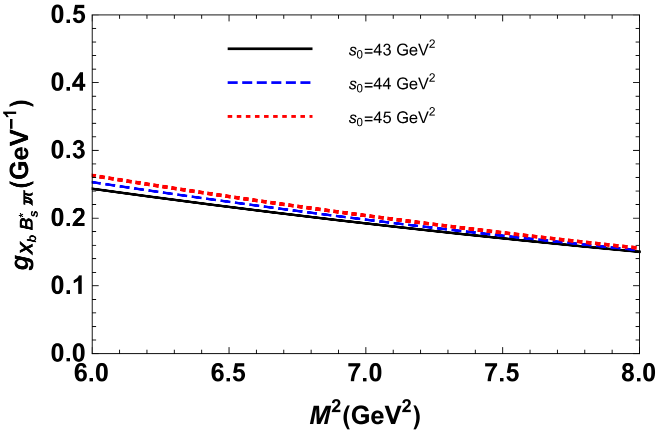

Fig. 4. Dependence of the strong coupling $g_{X_b B_s^* \pi}$ on the Borel parameter M^2.

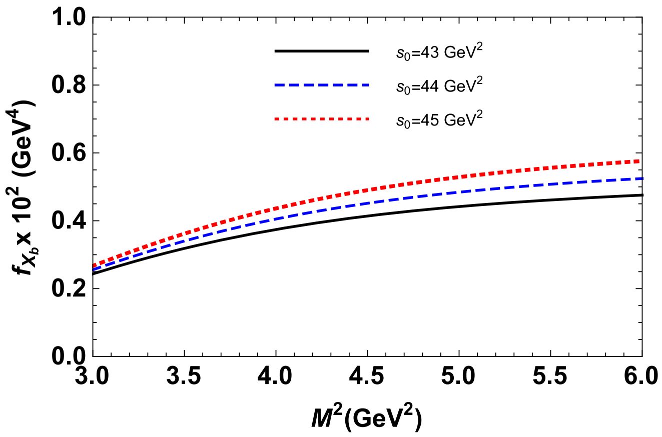

Fig. 3. The meson coupling f_{X_b} as a function of M^2.

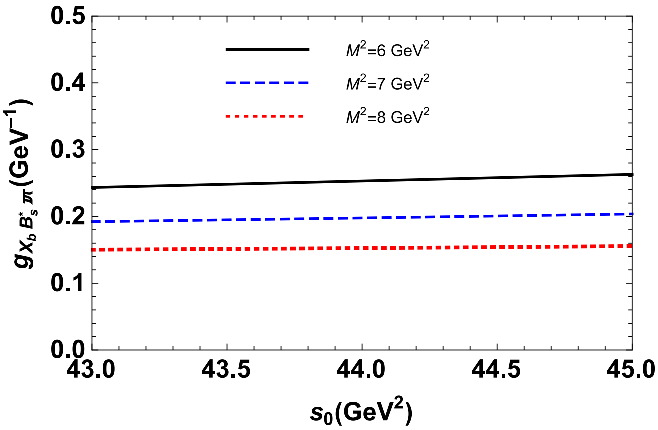

Fig. 5. The strong coupling $g_{X_b B_s^* \pi}$ as a function of s_0 at some fixed values of M^2.

In other words, treating $X(5568)$ as an axial-vector particle, we have to consider eq. (38) as the data of the D0 Collaboration. Then it becomes clear that our prediction for the mass of X_b differs from the result of the D0 Collaboration given by eq. (38): Even after taking into account errors of calculations it overshoots the corresponding experimental result.

In the evaluation of the strong coupling $g_{X_b B_s^* \pi}$ the window for the Borel parameter shifts towards larger values

$$6\,\mathrm{GeV}^2 \le M^2 \le 8\,\mathrm{GeV}^2, \tag{39}$$

whereas the range of s_0 remains unchanged. Results of our computations of the strong coupling $g_{X_b B_s^* \pi}$ and its dependence on M^2 and s_0 are depicted in figs. 4 and 5, respectively. One can see, that the strong coupling is sensitive to the choice of the Borel parameter and almost stable under the variation of s_0.

The coupling $g_{X_b B_s^* \pi}$ extracted from the sum rule eq. (30) reads

$$g_{X_b B_s^* \pi} = (0.20 \pm 0.05)\,\mathrm{GeV}^{-1}. \tag{40}$$

Carrying out a similar numerical analysis for the coupling $\widetilde{g}_{X_b B_s^* \pi}$ we find:

$$\widetilde{g}_{X_b B_s^* \pi} = (0.21 \pm 0.06)\,\mathrm{GeV}^{-1}. \tag{41}$$

The difference between predictions for the couplings obtained using the different Lorentz structures is rather small. We use its mean value $g \Rightarrow (g + \tilde{g})/2$ to evaluate the width of the decay $X_b \to B_s^* \pi^+$ process and get

$$\Gamma(X_b \to B_s^* \pi^+) = (20.2 \pm 6.1)\,\mathrm{MeV}, \tag{42}$$

whereas the experiment gives $\Gamma_{\mathrm{exp}}[X(5568) \to B_s^* \pi^+] = 21.9 \pm 6.4(\mathrm{stat})^{+5.0}_{-2.5}(\mathrm{syst})\,\mathrm{MeV}$.

In other words, the present investigation of the $X(5568)$ resonance and the comparison of the obtained predictions with data of the D0 Collaboration does not confirm an axial-vector nature of this diquark-antidiquark state, the mass $m[X_b(1^+)]$ parameter being the decisive argument in making this conclusion.

Within the QCD sum rule approach the mass of the $X(5568)$ state as an axial-vector particle was also investigated in ref. [26]. The result obtained in this paper for the mass of the exotic state X_b reads

$$m[X_b(1^+)] = 5.59 \pm 0.15\,\mathrm{GeV}. \tag{43}$$

As is seen, there is a discrepancy between our prediction for $m[X_b(1^+)]$ and the result of ref. [26]. The possible source of the dissonance are the working regions for the parameters s_0 and M^2 used there, which were extracted by means of the two-point spectral density that differs from $\rho^{\mathrm{QCD}}(s)$ derived in the present work, as has been noted above.

4 Concluding remarks

In the current work we have done the QCD sum rule analysis of the exotic X_b state by considering it as an axial-vector particle built of a diquark and antidiquark. We have computed the mass m_{X_b} and decay width of the process $X_b \to B_s^* \pi^+$, and compared our results with experimental data of the D0 Collaboration, as well as with the theoretical prediction for m_{X_b} made in ref. [26]. Our result for the mass of the axial-vector exotic state X_b exceeds the experimental data, whereas the decay width of the process $X_b \to B_s^* \pi^+$ calculated using two different structures in the correlation function is compatible with the experimental data. Despite this last fact, we make our conclusion relying mainly on the mass calculation: If the $X(5568)$ resonance exists and its parameters measured by the D0 Collaboration are correct, then the present analysis with high level of confidence excludes that it is an axial-vector diquark-antidiquark state with $J^P = 1^+$.

It is interesting to note that the same parameters were evaluated in refs. [23, 28] using the diquark-antidiquark model for the X_b state with quantum numbers $J^P = 0^+$. In these works a satisfactory agreement with the experimental data of the D0 Collaboration was found.

Further investigations are required to clarify the experimental situation and theoretical questions surrounding the $X(5568)$ resonance. But possible outputs of such studies justify the efforts paid for their realization, because the $X(5568)$ state is a very intriguing and interesting particle, supposedly composed of four-quarks of different flavor, and its investigation may shed light on many problems of hadron physics.

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Appendix A. The two-point spectral density

This appendix contains the results obtained for the two-point spectral density

$$\rho^{\mathrm{QCD}}(s) = \rho^{\mathrm{pert.}}(s) + \sum_{k=3}^{8} \rho_k(s) \tag{A.1}$$

used to calculate from the QCD sum rules the mass and meson coupling of the X_b state. In eqs. (A.1) and (A.2) by $\rho_k(s)$ we denote the nonperturbative contributions to $\rho^{\mathrm{QCD}}(s)$. The explicit expressions for $\rho^{\mathrm{pert.}}(s)$ and $\rho_k(s)$ are presented here as the integrals of the Feynman parameter z:

$$\rho^{\mathrm{pert.}}(s) = \frac{1}{12288\pi^6} \int_0^a \frac{\mathrm{d}z z^4}{(z-1)^3} \left[m_b^2 + s(z-1)\right]^3 [m_b^2 + 5s(z-1)],$$

$$\rho_3(s) = \frac{1}{128\pi^4} \int_0^a \frac{\mathrm{d}z z^2}{(z-1)^2} \left[m_b^2 + s(z-1)\right] \Big\{ 2\langle\overline{d}d\rangle m_b [m_b^2 + s(z-1)] + m_s(z-1)\left(\langle\overline{s}s\rangle - 2\langle\overline{u}u\rangle\right)\left[m_b^2 + 3s(z-1)\right] \Big\},$$

$$\rho_4(s) = \frac{1}{9216\pi^4} \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle \int_0^a \frac{\mathrm{d}z z^2}{(z-1)^3} \Big\{ m_b^4(13z^2 - 21z + 9) + 2m_b^2 s\left(23z^3 - 63z^2 + 58z - 18\right) + s^2(z-1)^3(32z-27) \Big\},$$

$$\rho_5(s) = \frac{m_0^2}{192\pi^4} \int_0^a \frac{\mathrm{d}z z}{(1-z)} \Big\{ 3m_b\langle\overline{d}d\rangle \left[m_b^2 + s(z-1)\right] + m_s(z-1)(\langle\overline{s}s\rangle - 3\langle\overline{u}u\rangle)\left[m_b^2 + 2s(z-1)\right] \Big\},$$

$$\rho_6(s) = \frac{1}{61440\pi^6} m_b^2 \langle g_s^3 G^3\rangle \int_0^a \mathrm{d}z \frac{z^5}{(1-z)^3} + \frac{1}{1296\pi^4} \times \int_0^a \mathrm{d}z \Big\{ g_s^2 \langle\overline{d}d\rangle^2 z \left[m_b^2 + 2s(z-1)\right] + 54\pi^2 m_b m_s \langle\overline{d}d\rangle(\langle\overline{s}s\rangle - 2\langle\overline{u}u\rangle) + z\left[g_s^2(\langle\overline{u}u\rangle^2 + \langle\overline{s}s\rangle^2) + 108\pi^2\langle\overline{s}s\rangle\langle\overline{u}u\rangle\right]\left[m_b^2 + 2s(z-1)\right] \Big\},$$

$$\rho_7(s) = \frac{1}{1152\pi^2} \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle \int_0^a \mathrm{d}z \Big\{ 8m_b\langle\overline{d}d\rangle + m_s\left[3z\langle\overline{s}s\rangle + 2\langle\overline{u}u\rangle(1-4z)\right] \Big\},$$

$$\rho_8(s) = \frac{11}{73728\pi^2} \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle^2 \int_0^a z\mathrm{d}z + \frac{1}{24\pi^2} m_0^2 \langle\overline{s}s\rangle\langle\overline{u}u\rangle \int_0^a (z-1)\mathrm{d}z, \tag{A.2}$$

where $a = 1 - m_b^2/s$.

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