Research Article

Impact of Scalar Leptoquarks on Heavy Baryonic Decays

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We present a study on the impact of scalar leptoquarks on the semileptonic decays of $\Lambda_b$, $\Sigma_b$, and $\Xi_b$. To this end, we calculate the differential branching ratio and lepton forward-backward asymmetry defining the processes $\Lambda_b \to \Lambda \ell^+ \ell^-$, $\Sigma_b \to \Sigma \ell^+ \ell^-$, and $\Xi_b \to \Xi \ell^+ \ell^-$, with $\ell$ being $\mu$ or $\tau$, using the form factors calculated via light cone QCD in full theory. In calculations, the errors of form factors are taken into account. We compare the results obtained in leptoquark model with those of the standard model as well as the existing lattice QCD predictions and experimental data.

1. Introduction

The physics of transitions based on $b \to s \ell^+ \ell^-$ at quark level constitutes one of the main directions of the research in high energy and particle physics both theoretically and experimentally as new physics effects can contribute to such decay channels. The flavor changing neutral current (FCNC) transitions of $\Lambda_b \to \Lambda \ell^+ \ell^-$, $\Sigma_b \to \Sigma \ell^+ \ell^-$, and $\Xi_b \to \Xi \ell^+ \ell^-$ are among important baryonic decay channels that can be used as sensitive probes to indirectly search for new physics contributions. Particularly, the rare $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay channel has been in the focus of much attention in recent years both theoretically and experimentally. The first measurement on the $\Lambda_b \to \Lambda \ell^+ \ell^-$ process has been reported by the CDF Collaboration [1] with 24 signal events and a statistical significance of 5.8 Gaussian standard deviations. Using the $p\bar{p}$ collisions data samples corresponding to $6.8 \text{ fb}^{-1}$ and $\sqrt{s} = 1.96 \text{ TeV}$ collected by the CDF II detector, the differential branching ratio for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay channel has been measured to be $\text{d}Br(\Lambda_b^0 \to \Lambda \mu^+ \mu^-)/d\!q^2 = [1.73 \pm 0.42{\text{(stat)}} \pm 0.55{\text{(syst)}}] \times 10^{-6}$ [1]. The differential branching fraction of $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ decay channel has also been measured as $\text{d}Br(\Lambda_b^0 \to \Lambda \mu^+ \mu^-)/d\!q^2 = \left(1.18^{+0.09}_{-0.08} \pm 0.03 \pm 0.27\right) \times 10^{-7} \text{ GeV}^2/\text{c}^4$ at $15 \text{ GeV}^2/\text{c}^4 \leq q^2 \leq 20 \text{ GeV}^2/\text{c}^4$ region by the LHCb Collaboration [2]. The LHCb Collaboration has also measured the lepton forward-backward asymmetries associated with this transition as $A_{FB}^\ell = -0.05 \pm 0.09{\text{(stat)}} \pm 0.03{\text{(syst)}}$ at $15 \text{ GeV}^2/\text{c}^4 \leq q^2 \leq 20 \text{ GeV}^2/\text{c}^4$ region [2]. The order of branching ratio in $\Lambda_b \to \Lambda \ell^+ \ell^-$ and $\Sigma_b \to \Sigma \ell^+ \ell^-$, and $\Xi_b \to \Xi \ell^+ \ell^-$ are all accessible at LHC (for details see [3–7]). We hope that with the RUN II data at the center of mass energy 13 TeV it will be possible to measure different physical quantities related to these FCNC loop level rare transitions in near future.

The LHC RUN II may provide opportunities to search for various new physics scenarios. One of the important new physics models that has been proposed to overcome the problems of some inconsistencies between the SM predictions and experimental data is the leptoquark (LQ) model. Hereafter, by LQ model we mean a minimal renormalizable scalar leptoquark model which will be explained in some details in next section. As an example for the LHC constraints and prospects for scalar leptoquarks explaining the $\bar{B} \to D^{(*)}\tau\bar{\nu}$ anomaly see [8]. LQs are hypothetical color triplet bosons that couple to leptons and quarks [9]. LQs carry both baryon (B) and lepton (L) quantum numbers with color and electric charges. The spin number of a leptoquark state can be 0 or 1, corresponding to a scalar leptoquark or vector leptoquark. If the leptoquarks violate both the baryon and lepton numbers,
they are generally considered to be heavy particles at the level of $\mathcal{O}(10^{15})$ GeV in order to prevent the proton decay. For more detailed information about leptoquark models and the recent experimental and theoretical progress, see [10–30].

In the light of progress about LQs, we calculate the differential branching ratio and lepton forward-backward asymmetry corresponding to $\Lambda_b \to \Lambda e^+ e^-$, $\Sigma_b \to \Sigma e^+ e^-$, and $\Xi_b \to \Xi e^+ e^-$ processes in a scalar LQ model. In the calculations, we use the form factors as the main inputs calculated from the light cone QCD sum rules in full theory. We also encounter the errors of the form factors to the results with the available lattice predictions and experimental data.

The outline of this article is as follows. In next section, we present the transition amplitude and matrix elements defining the above transitions. In Section 3, we present the transition amplitude and matrix elements defining the effective Hamiltonian responsible for the transitions under consideration both in the SM and LQ models.

## 2. The Effective Hamiltonian and Wilson Coefficients

At the quark level the effective Hamiltonian, defining the above-mentioned $b \to s e^+ e^-$ based transitions, in terms of Wilson coefficients and different operators in SM is generally defined as [32, 33]

$$ R_{SM}^{\text{eff}} = \frac{G_F^2 \alpha_{em} V_{tb} V_{ts}^*}{2 \sqrt{2} \pi} \left[ C_9^{\text{eff}} \bar{y}_\mu (1 - y_5) b \bar{e} \gamma_\mu e + C_9^{\text{eff}} \bar{y}_\mu (1 + y_5) b \bar{e} \gamma_\mu y_5 e + C_9^{\text{eff}} \bar{y}_\mu (1 - y_5) b \bar{e} y_\mu y_5 e + C_9^{\text{eff}} \bar{y}_\mu (1 + y_5) b \bar{e} y_\mu y_5 e - 2m_b C_9^{\text{eff}} \frac{1}{q^2} \bar{y}_\mu (1 - y_5) b \bar{e} y_\mu e - 2m_b C_9^{\text{eff}} \frac{1}{q^2} \bar{y}_\mu (1 + y_5) b \bar{e} y_\mu e \right], $$

where $G_F$ is the Fermi weak coupling constant, $\alpha_{em}$ is the fine structure constant at Z mass scale, $V_{tb}$ and $V_{ts}$ are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $C_9^{(l)}$, $C_{10}^{(l)}$, and $C_9^{(l)}$ are the SM Wilson coefficients, and $q^2$ is the transferred momentum squared. Here the superscript “eff” refers to the shifts in the corresponding coefficients due to the effects of four-quark operators at large $q^2$. The primed coefficients are ignored since the Hamiltonian does not receive any contribution from the corresponding operators in the SM. We collect the explicit expressions of the Wilson coefficients $C_9^{\text{eff}}$, $C_{10}^{\text{eff}}$, and $C_9^{\text{eff}}$ in Appendix A.

In the light of progress about LQs, we calculate the differential branching ratio and lepton forward-backward asymmetry corresponding to $\Lambda_b \to \Lambda e^+ e^-$, $\Sigma_b \to \Sigma e^+ e^-$, and $\Xi_b \to \Xi e^+ e^-$ processes in a scalar LQ model. In the calculations, we use the form factors as the main inputs calculated from the light cone QCD sum rules in full theory. We also encounter the errors of the form factors to the results with the available lattice predictions and experimental data.

The outline of this article is as follows. In next section, we present the transition amplitude and matrix elements defining the above transitions. In Section 3, we present the transition amplitude and matrix elements defining the effective Hamiltonian responsible for the transitions under consideration both in the SM and LQ models.
contribution to the branching ratio does not exceed the experimental result. Here
\[
0 \leq \left| \frac{\lambda_{c}^{13} \lambda_{s}^{12*}}{M_{V}^{2}} \right| = \left| \frac{\lambda_{s}^{13} \lambda_{c}^{12*}}{M_{V}^{2}} \right| \leq 5 \times 10^{-9} \text{ GeV}^{-2},
\] obtained via the fitting of the model parameters to the $B_{s} \to \mu^{+} \mu^{-}$ data [34]. In (5) we assumed that the contributions of the two components $Y$ and $V$ are equal.

### 3. Transition Amplitude and Matrix Elements

Generally, the amplitude of the transition responsible for the $\Lambda_{b} \to \Lambda \ell^{+} \ell^{-}$, $\Sigma_{b} \to \Sigma \ell^{+} \ell^{-}$, and $\Xi_{b} \to \Xi \ell^{+} \ell^{-}$ baryonic decays is provided with sandwiching the effective Hamiltonian between the initial and final baryonic states,
\[
\mathcal{M}_{\Lambda_{b} \to \Lambda \ell^{+} \ell^{-}} = \langle \mathcal{R} (p) | \mathcal{H}_{\text{eff}} | \mathcal{Q}_{3} (p + q, s) \rangle,
\] where $\mathcal{R}$ represents $\Lambda$, $\Sigma$, and $\Xi$ baryons and $Q$ corresponds to $b$ quark. To get the transition amplitude, we need to consider the following transition matrix elements parametrized in terms of twelve form factors in full QCD, that is, without any expansion in the heavy quark mass or large hadron energies:
\[
\langle \mathcal{R} (p) | \mathcal{H}_{\text{eff}} | \mathcal{Q}_{3} (p + q, s) \rangle = \mathcal{M}_{\text{tot}} (p, q, s),
\]
\[
\mathcal{M}_{\text{tot}} (p, q, s) = \mathcal{M}_{\text{SM}} (p, q, s) + \mathcal{M}_{\text{LQ}} (p, q, s),
\]
\[
\mathcal{M}_{\text{SM}} (p, q, s) = \frac{G_{F} \alpha_{s} \Lambda_{V} V_{tb}^{*}}{2\sqrt{2} \pi} \left\{ \left[ \Pi_{\beta} (p) \right] \cdot \left( \mathcal{F}_{1}^{\text{SM}} R + \mathcal{F}_{1}^{\text{SM}} L \right) \right\}
\]
\[
+ q^{\mu} \left[ \mathcal{F}_{2}^{\text{SM}} R + \mathcal{F}_{2}^{\text{SM}} L \right] u_{\beta_{Q}} (p + q, s) \left( \bar{\nu} \gamma_{\mu} \nu \right),
\]
\[
\mathcal{M}_{\text{LQ}} (p, q, s) = \frac{G_{F} \alpha_{s} \Lambda_{V} V_{tb}^{*}}{2\sqrt{2} \pi} \left\{ \left[ \Pi_{\beta} (p) \right] \cdot \left( \mathcal{F}_{1}^{\text{tot}} R + \mathcal{F}_{1}^{\text{tot}} L \right) \right\}
\]
\[
+ q^{\mu} \left[ \mathcal{F}_{2}^{\text{tot}} R + \mathcal{F}_{2}^{\text{tot}} L \right] u_{\beta_{Q}} (p + q, s) \left( \bar{\nu} \gamma_{\mu} \nu \right),
\]
where $R = (1 + \gamma_{5})/2$ and $L = (1 - \gamma_{5})/2$ and the calligraphic coefficients are collected in Appendix B.

### 4. Physical Observables

In this section we would like to calculate some physical observables such as the differential decay width, the differential branching ratio, and the lepton forward-backward asymmetry for the considered decay channels.

#### 4.1. The Differential Decay Width

Using the decay amplitudes and transition matrix elements in terms of form factors, we find the differential decay rate defining the transitions under consideration in the LQ model as
\[
\frac{d^{2} \Gamma (z, \bar{z})}{d s d z} = \frac{G_{F} \alpha_{s} \Lambda_{V} V_{tb}^{*}}{16384 \pi^{5}} \left| V_{tb} V_{1 s}^{*} \right|^{2}
\]
\[
\cdot \sqrt{\lambda} (1, r, s) \left[ \mathcal{T}_{1}^{\text{tot}} (s) \right] \mathcal{T}_{1}^{\text{tot}} (s) z + \mathcal{T}_{2}^{\text{tot}} (s) z^{2},
\]
where $v = \sqrt{1 - 4m_\ell^2/q^2}$ is the lepton velocity, $\lambda = \lambda(1, r, \tilde{s}) = (1-r-\tilde{s})^2 - 4r\tilde{s}$ is the usual triangle function, $\tilde{s} = q^2/m_\ell^2 R_Q$ for the decay channels under consideration. For this aim, we shall remark that the above-mentioned values/intervals for these coefficients denote the maximum and minimum values obtained varying intervals for these coefficients.

Table 2: The values of quark masses in $\overline{MS}$ scheme [9].

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Masses in $\overline{MS}$ scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t$</td>
<td>$(1.275 \pm 0.025)$ GeV</td>
</tr>
<tr>
<td>$m_b$</td>
<td>$(4.18 \pm 0.03)$ GeV</td>
</tr>
<tr>
<td>$m_t$</td>
<td>$160_{-4.8}^{+4.4}$ GeV</td>
</tr>
</tbody>
</table>

4.2. The Differential Branching Ratio. Using the expression of the differential decay width in this subsection, we numerically analyze the differential branching ratio in terms of $q^2$ for the decay channels under consideration. For this aim, we present the values of some input parameters and the quark masses in $\overline{MS}$ scheme used in the numerical analysis in Tables 1 and 2 [9]. Using the numerical values in these tables and the expressions presented in Appendix A, we find the values/intervals $C_{\gamma}^{\text{eff}} = -0.295$, $C_{\gamma}^{\text{eff}} = [1.573, 6.625]$, $C_{\gamma} = -4.260$, $C_{\gamma}^{\text{tot}} = [2.793, 4.394]$, $C_{\gamma}^{\text{eff,tot}} = [0, 1.586]$, $C_{\gamma}^{\text{tot}} = [-5.846, -4.260]$, and $C_{\gamma}^{\text{tot}} = [-1.586, 0]$ for the corresponding Wilson coefficients. Since $C_{\gamma}^{\text{eff(tot)}}$ depend on $q^2$, the above intervals for these coefficients denote the maximum and minimum values obtained varying $q^2$ in the physical region, that is, $[0-20]$ GeV$^2$. In the case of coefficients with label ”tot” the above intervals are obtained considering the intervals for related parameters in (5). Note that we will use directly the expressions of the Wilson coefficients in the numerical analyses instead of the above-mentioned values/intervals. We shall remark that the above-mentioned values/intervals for $C_{\gamma}^{\text{eff}}$, $C_{\gamma}^{\text{eff}}$, and $C_{\gamma}^{\text{tot}}$ are consistent with the ones obtained in [38–43] for Wilson coefficients using the global fits to $b \to s\ell^+\ell^-$ data. We would also like to compare the intervals for four Wilson coefficients $C_{\gamma}^{\text{eff,tot}}$, $C_{\gamma}^{\text{eff,tot}}$, $C_{\gamma}^{\text{tot}}$, and $C_{\gamma}^{\text{tot}}$ which are relevant to the LQ model with the values obtained in [31] from experimental data on observables of $\Lambda_b \to \Lambda\mu^+\mu^-$ transition in the SM and LQ models. The experimental data are taken from the LHCb Collaboration [2]. The lattice predictions are borrowed from [31]. The vertical shaded bands indicate the charmonia veto regions.

Figure 1: The dependence of the differential branching ratio on $q^2$ for the $\Lambda_b \to \Lambda\mu^+\mu^-$ transition in the SM and LQ models. The experimental data are taken from the LHCb Collaboration [2]. The lattice predictions are borrowed from [31]. The vertical shaded bands indicate the charmonia veto regions.
Figure 2: The dependence of the differential branching ratio on $q^2$ for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ transition in the SM and LQ models. The vertical shaded band indicates the charmonia veto region.

Figure 3: The dependence of the differential branching ratio on $q^2$ for the $\Sigma_b \to \Sigma \mu^+ \mu^-$ transition in the SM and LQ models. The vertical shaded bands indicate the charmonia veto regions.

Figure 4: The dependence of the differential branching ratio on $q^2$ for the $\Xi_b \to \Xi \mu^+ \mu^-$ transition in the SM and LQ models. The vertical shaded band indicates the charmonia veto region.

Figure 5: The dependence of the differential branching ratio on $q^2$ for the $\Xi_b \to \Xi \tau^+ \tau^-$ transition in the SM and LQ models. The vertical shaded bands indicate the charmonia veto regions.

4.3. The Lepton Forward-Backward Asymmetry. In this subsection, we present the results of the lepton forward-backward asymmetry ($A_{FB}$) which is one of useful observables to search for NP effects. This quantity is defined as

$$A_{FB}(\bar{s}) = \frac{\int_{0}^{1} (d^2\Gamma/ds\,dz) (z, \bar{s}) \,dz - \int_{-1}^{0} (d^2\Gamma/ds\,dz) (z, \bar{s}) \,dz}{\int_{0}^{1} (d^2\Gamma/ds\,dz) (z, \bar{s}) \,dz + \int_{-1}^{0} (d^2\Gamma/ds\,dz) (z, \bar{s}) \,dz}. \tag{10}$$

In order to see how predictions of LQ scenario deviate from those of the SM, we plot the dependence of the lepton forward-backward asymmetry on $q^2$ for the channels under discussion in Figures 7–12. In Figure 7, we also present the measured values of the leptonic forward-backward-asymmetries by the LHCb Collaboration [2] as well as the lattice QCD predictions [31] in the $\Lambda_b \to \Lambda \mu^+ \mu^-$ channel.

From these figures, we read that the LQ model. The LQ model bands are wider and somewhat show considerable discrepancies from the SM predictions for all channels roughly at whole physical region of $q^2$;

(ii) the SM band for the differential branching fraction in $\Lambda_b \to \Lambda \mu^+ \mu^-$ channel roughly coincides with all the lattice predictions borrowed from [31]. This band also defines all the experimental data provided by the LHCb Collaboration except that in the interval $18 \text{GeV}^2/c^4 \leq q^2 \leq 20 \text{GeV}^2/c^4$, which can not be described by the SM. This datum coincides with the LQ band. As is also seen from this figure the lattice QCD predictions on the differential branching fraction in $\Lambda_b \to \Lambda \mu^+ \mu^-$ channel show considerable discrepancies with the experimental data in the interval $15 \text{GeV}^2/c^4 \leq q^2 \leq 20 \text{GeV}^2/c^4$.

SM with errors
LQ model with errors
Figure 6: The dependence of the differential branching ratio on $q^2$ for the $\Sigma_b \rightarrow \Xi^+ \tau^-\tau^+$ transition in the SM and LQ models. The vertical shaded band indicates the charmonia veto region.

(i) in all decay channels the LQ model predictions demonstrate considerable discrepancies from the SM predictions;
(ii) the SM band on the lepton forward-backward asymmetry in $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ channel coincides with the existing lattice QCD predictions borrowed from [31];
(iii) ignoring from the small intersection of the SM narrow bands with errors of the experimental data at very low and high values of $q^2$, the LQ model, against the SM, can describe all data available in $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ channel. The lattice QCD predictions in this channel also show sizable differences with the experimental data.

5. Conclusion

In the present work, we have performed a comprehensive analysis of the semileptonic $\Lambda_b \rightarrow \Lambda \ell^-\ell^+$, $\Sigma_b \rightarrow \Sigma^+\ell^-\ell^+$, and $\Xi_b \rightarrow \Xi^+\ell^-\ell^+$ rare processes in the SM as well as the scalar leptoquark model. Using the parametrization of the matrix elements in terms of form factors calculated via light cone QCD sum rules in the full theory, we calculated the differential decay width and numerically analyzed the differential branching fraction in terms of $q^2$ in different heavy baryonic decay channels for both $\mu$ and $\tau$ leptons in both scenarios. We compared the predictions of the LQ model on the considered physical observables with those of the SM and the existing lattice QCD predictions as well as experimental data in $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ channel. We observed that the predictions of the LQ model in all channels show considerable discrepancies with those of the SM on both the differential decay width and lepton forward-backward asymmetry. The SM results for both the observables considered in the present study are consistent with the existing predictions of lattice QCD. Except the interval $18 \text{ GeV}^2/c^4 \leq q^2 \leq 20 \text{ GeV}^2/c^4$, the SM band describes the existing experimental data on the differential branching ratio in $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ transition. The datum in $18 \text{ GeV}^2/c^4 \leq q^2 \leq 20 \text{ GeV}^2/c^4$ coincides with the LQ model prediction.

In the case of lepton forward-backward asymmetry, the SM, overall, can not describe the experimental data existing in $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ channel, while the LQ model band coincides with the experimental data.

More experimental data in $\Lambda_b \rightarrow \Lambda \tau^+\tau^-$ as well as $\Sigma_b \rightarrow \Sigma^+\ell^-\ell^+$ and $\Xi_b \rightarrow \Xi^+\ell^-\ell^+$ with both leptons are needed to compare with the theoretical predictions. We hope that, with the RUN II data, it will be possible to measure different physical quantities related to such FCNC transitions at LHCb in near future. Comparison of the future experimental data with the theoretical predictions on different physical quantities in various decay channels can help us

Figure 7: The dependence of $\mathcal{A}_{FB}$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ transition in the SM and LQ models. The experimental data are taken from the LHCb Collaboration [2]. The lattice predictions are borrowed from [31]. The vertical shaded bands indicate the charmonia veto regions.

Figure 8: The dependence of $\mathcal{A}_{FB}$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda \tau^+\tau^-$ transition in the SM and LQ models. The vertical shaded band indicates the charmonia veto region.
better explain some anomalies between the SM predictions and the experimental data. Any sizable discrepancy between the theoretical predictions on physical observables with the experimental data can be considered as an indication of new physics effects and may help us in the course of searching for the new particles like leptoquarks.

**Note Added.** When preparing this work we noticed that a part of our work, namely, the \( \Lambda_b \to \Lambda \ell^+ \ell^- \) channel has been investigated in [34, 45] within the same framework. In these studies the authors use the form factors, as the main inputs, calculated in heavy quark effective theory while we use the form factors calculated via light cone QCD sum rules in full theory.

**Appendix**

**A. The Wilson Coefficients**

The Wilson coefficient \( C_7^{\text{eff}} \) in leading logarithm approximation in the SM is written by [46–49]

\[
C_7^{\text{eff}} (\mu_W) = \eta^{16/23} C_7 (\mu_W) \\
+ \frac{8}{3} \left( \eta^{14/23} - \eta^{16/23} \right) C_8 (\mu_W) \\
+ C_2 (\mu_W) \sum_{i=1}^{8} h_i \eta^i,
\]  

where

\[
C_7 (\mu_W) = -\frac{1}{2} D_0 (x_t), \\
C_8 (\mu_W) = -\frac{1}{2} D_0' (x_t),
\]
\[ C_2 (\mu_W) = 1. \] (A.2)

The functions \( D'_0(x_i) \) and \( E'_0(x_i) \) with \( x_i = m_t^2/m_W^2 \) are given as

\[
D'_0(x_i) = -\frac{8x_i^3 + 5x_i^2 - 7x_i}{12(1-x_i)^3} + \frac{x_i^2(2-3x_i)}{2(1-x_i)^3} \ln x_i, \\
E'_0(x_i) = -\frac{x_i(x_i^2 - 5x_i - 2)}{4(1-x_i)^3} + \frac{3x_i^2}{2(1-x_i)^2} \ln x_i. \] (A.3)

The Wilson coefficient \( C_{9}^{\text{eff}} \) in SM is given by [47, 48]

\[ C_{9}^{\text{eff}} (\hat{s}') = C_{9}^{\text{NDR}} \eta (\hat{s}') + h(z, \hat{s}') (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2} h(1, \hat{s}') (4C_3 + 4C_4 + 3C_5 + C_6) - \frac{1}{2} h(0, \hat{s}') (C_3 + 3C_4) + \frac{2}{9} (3C_3 + 4C_3 + 3C_5 + C_6), \] (A.7)

where \( \hat{s}' = q^2/m_b^2 \) with \( 4m_b^2 \leq q^2 \leq (m_{\tilde{W}_L} - m_{\tilde{b}})^2 \). \( C_{9}^{\text{NDR}} \) in the naive dimensional regularization (NDR) scheme is written as

\[ C_{9}^{\text{NDR}} = P_0^{\text{NDR}} + \frac{Y}{\sin^2 \theta_W} - 4Z + P_b E_i \] (A.8)

where \( P_0^{\text{NDR}} = 2.60 \pm 0.25, \, \sin^2 \theta_W = 0.23, \, Y = 0.98, \) and \( Z = 0.679 [47–49] \). The last term in (A.8) is ignored due to the negligible value of \( P_b \). In (A.7), \( \eta (\hat{s}') \) is given as

\[ \eta (\hat{s}') = 1 + \frac{\alpha_s (\mu_W)}{\pi} \omega (\hat{s}'), \] (A.9)

with

\[
\omega (\hat{s}') = \frac{2}{9} \pi^2 - \frac{4}{3} \text{Li}_2 (\hat{s}') - \frac{2}{3} \ln \hat{s}' \ln (1-\hat{s}') - \frac{5 + 4 \hat{s}'}{3 (1 + 2 \hat{s}')} \ln (1-\hat{s}').
\]

The parameter \( \eta \) in (A.1) is defined as

\[ \eta = \frac{\alpha_s (\mu_W)}{\alpha_s (\mu_b)}, \] (A.4)

with

\[ \alpha_s (x) = \frac{\alpha_s (m_Z)}{1 - \beta_0 (\alpha_s (m_Z)/2 \pi) \ln (m_Z/x)}. \] (A.5)

where \( \alpha_s (m_Z) = 0.118 \) and \( \beta_0 = 23/3 \). The coefficients \( h_i \) and \( a_i \) in (A.1) are also written by [47, 48]

\[
h_i = \begin{pmatrix} 2.2996, & -1.0880, & -\frac{3}{7}, & -\frac{1}{14}, & -0.6494, & -0.0380, & -0.0186, & -0.0057 \end{pmatrix},
\]

\[
a_i = \begin{pmatrix} 14 & 16 & 6 & 12 \end{pmatrix} ^{T} \begin{pmatrix} 0.4086, & -0.4230, & -0.8994, & 0.1456 \end{pmatrix}.
\]

The function \( h(y, \hat{s}') \) is written as

\[
h(y, \hat{s}') = -\frac{8}{9} \ln \frac{m_b}{\mu_b} - \frac{8}{9} \ln y + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) |1 - x|^{1/2} \left\{ \ln \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} - i \pi \right\}, \quad \text{for } x \equiv \frac{4x^2}{\hat{s}'} < 1,
\]

\[
2 \left| \arctan \frac{1}{\sqrt{x - 1}} \right|, \quad \text{for } x \equiv \frac{4x^2}{\hat{s}'} > 1,
\]

where \( y = 1 \) or \( y = z = m_{\tilde{c}}/m_b \) and

\[
h(0, \hat{s}') = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu_b} - \frac{4}{9} \ln z' + \frac{4}{9} i \pi.
\]

The coefficients \( C_j \) \( (j = 1, \ldots, 6) \) at \( \mu_b = 5 \) GeV scale are also written as [49]

\[ C_j = \sum_{i=1}^{8} k_{ji} \eta^{a_i} \quad (j = 1, \ldots, 6), \] (A.13)

where \( k_{ji} \) are given as

\[
k_{11} = \begin{pmatrix} 0, & 0, & 1, & -\frac{1}{2}, & 0, & 0, & 0, & 0 \end{pmatrix},
\]

\[
k_{21} = \begin{pmatrix} 0, & 0, & 1, & \frac{1}{2}, & 0, & 0, & 0, & 0 \end{pmatrix}.
\]
Considering the resonances from $J/\psi$ family, we divide the allowed physical region into the following three regions in the case of the electron and muon as final leptons:

- **Region I**: $4m_0^2 \leq q^2 \leq (m_{J/\psi(1S)} - 0.02)^2$.
- **Region II**: $(m_{J/\psi(1S)} + 0.02)^2 \leq q^2 \leq (m_{J/\psi(2S)} - 0.02)^2$.
- **Region III**: $(m_{J/\psi(2S)} + 0.02)^2 \leq q^2 \leq (m_{J/\psi(3S)} - m_{J/\psi(2S)})^2$.

In the case of $\tau$, we have the following two regions:

- **Region I**: $4m_0^2 \leq q^2 \leq (m_{J/\psi(2S)} - 0.02)^2$.
- **Region II**: $(m_{J/\psi(2S)} + 0.02)^2 \leq q^2 \leq (m_{J/\psi(3S)} - m_{J/\psi(2S)})^2$.

The Wilson coefficient $C_{10}$ in the SM is given as

$$C_{10} = \frac{-Y}{\sin^2 \theta_W}. \quad (A.15)$$

### B. The Functions Used in Transition Amplitudes and Differential Decay Rate

The calligraphic coefficients used in the transition amplitudes of the considered processes are find as

\[
\mathcal{A}_1 = f_1 C_{y_{10}}^\text{eff} - g_1 C_{y_{10}}^\text{eff*} - 2m_{\psi} \frac{1}{q^2} \left[ f_1 C_{y_{10}}^\text{eff} + g_1 C_{y_{10}}^\text{eff*} \right],
\]

\[
\mathcal{A}_2 = \mathcal{A}_1 \left( 1 \to 2 \right),
\]

\[
\mathcal{A}_3 = \mathcal{A}_1 \left( 1 \to 3 \right),
\]

\[
\mathcal{B}_1 = f_1 C_{y_{10}}^\text{eff} + g_1 C_{y_{10}}^\text{eff*} - 2m_{l_0} q^2 \left[ f_1 C_{y_{10}}^\text{eff*} - g_1 C_{y_{10}}^\text{eff} \right].
\]

The functions $\mathcal{F}_0^\text{tot}(\psi)$, $\mathcal{F}_1^\text{tot}(\psi)$, and $\mathcal{F}_2^\text{tot}(\psi)$ in the differential decay width are given as

\[
\mathcal{F}_0^\text{tot}(\psi) = 32 m_{J/\psi(1S)}^4 \left( 1 + r - \bar{s} \right) \left( |\mathcal{D}|^2 + |\mathcal{E}|^2 \right)
\]

\[
+ 64 m_{J/\psi(1S)}^2 (1 - r - \bar{s}) \Re \left[ \mathcal{D}^* \mathcal{G} + \mathcal{D} \mathcal{G}^* \right]
\]

\[
+ 64 m_{J/\psi(1S)}^2 \sqrt{r} \left( 6 m_{\psi}^2 - m_{J/\psi(2S)}^2 \right) \Re \left[ \mathcal{D}^* \mathcal{E} \right]
\]

\[
+ 64 m_{J/\psi(1S)}^2 \sqrt{r} \left( 2 m_{J/\psi(2S)}^2 \Re \left[ \mathcal{G}^* \mathcal{E} \right] + (1 - r + \bar{s})
\]

\[
\cdot \Re \left[ \mathcal{D}^* \mathcal{D} + \mathcal{E}^* \mathcal{G} \right] \right] + 32 m_{J/\psi(1S)}^2 \left( 2 m_{\psi}^2 + m_{J/\psi(2S)}^2 \right)
\]

\[
\cdot \left\{ (1 - r + \bar{s}) m_{J/\psi(1S)} \sqrt{r} \Re \left[ \mathcal{D}^* \mathcal{D} + \mathcal{E}^* \mathcal{G} \right] \right\}
\]

\[
- m_{J/\psi(1S)} (1 - r - \bar{s}) \Re \left[ \mathcal{D}^* \mathcal{D} + \mathcal{E}^* \mathcal{G} \right]
\]

\[
- 2 \sqrt{r} \left( \Re \left[ \mathcal{D}^* \mathcal{D} + m_{J/\psi(2S)}^2 \Re \left[ \mathcal{D}^* \mathcal{D} \right] \right] \right)
\]

\[
+ 8 m_{J/\psi(1S)}^2 \left\{ 4 m_{\psi}^2 (1 + r - \bar{s}) + m_{J/\psi(2S)}^2 \left[ (1 - r)^2 - \bar{s}^2 \right] \right\}
\]
\[ \left( |\mathcal{M}_1|^2 + |\mathcal{B}_1|^2 \right) + 8m_{\bar{q}_0}^4 \left[ 4m_F^2 \left( 1 + r - s \right) \tilde{s} \right] + m_{\bar{q}_0}^2 \left( 1 - r \right)^2 - s^2 \left( 1 \right) \left| \mathcal{A}_1 \right|^2 + \left| \mathcal{B}_2 \right|^2 \]

\[ - 8m_{\bar{q}_0}^2 \left[ 4m_F^2 \left( 1 + r - s \right) - m_{\bar{q}_0}^2 \left( 1 - r \right)^2 - s^2 \right] \left( 1 \right) \left( 1 \right) \left| \mathcal{A}_1 \right|^2 + \left| \mathcal{B}_2 \right|^2 \]

\[ \cdot \left( |\mathcal{A}_1|^2 + |\mathcal{B}_1|^2 \right) \]

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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