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A Dynamic Model of a Nonlife Insurance Company

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Abstract

An analysis of optimal investments in technical capacity, goodwill and financial portfolios to maximize the long term profit of a nonlife company is conducted using Optimal Control Theory. Technical capacity affects both the claim ratio and the number of policies while goodwill affects only the number of policies. Financial portfolio is assumed to be optimally managed hence it is not analysed.

Mathematically technical capacity (T) is assumed to obey the following differential equation;

\[ T' = dT / dt = q - \delta T \]

Therefore, technical capacity increases by the investments (q) made in underwriting personnel, their education, software and hardware for technical analysis and decreases by the obsolescence of technical methods and the depreciation of hardware at a rate \( \delta \).

Goodwill (G) is assumed by to obey the following differential equation;

\[ G' = a - \eta G \]

where, (a) denotes the investment made in quality and advertising and \( \eta \) denotes the exponential rate of decline of goodwill.

In a competitive insurance market (where price is constant), the profit of a company at any time t is expressed as;

\[ \Pi = \pi \cdot N(G,T) - \pi \cdot N(G,T) \cdot H(T) - q - w(a) \]

N represents number of policies sold. H(T) represents the claim ratio. q and a represents the investments made in technical capacity and goodwill respectively. w(a) represents the convex cost function of goodwill expenditures. The objective of the company is to maximize discounted profits over the infinite horizon. Analysis of the differential equations representing the necessary conditions for the solution of the problem gives the following results:

1. The company with lower than required goodwill at the beginning should increase the expenditures in both goodwill and the technical capacity at an increasing rate if the impact of technical capacity on claim ratio is greater than its impact on the number of customers.

2. The company should first invest heavily in goodwill to increase the number of customers and then increase the investment in technical capacity to decrease the claim ratio if impact of technical capacity on claim ratio is less than its impact on the number of customers.

3. A sudden drop in the level of goodwill can be increased by investment quality and advertising. Reinsurance is not included in the model since this function is a partnership function. Other expenses are assumed to be constant and hence they do affect the solution.

Keywords : Nonlife Insurance, Optimal Control, Technical Investment.

JEL Classification Codes : C61, G22.